BULLETIN

OF THE

AMERICAN MATHEMATICAL SOCIETY

EDITED BY

EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

HAROLD T. DAVIS HARRY W. KUHN
THEOPHIL H. HILDEBRANDT DERRICK N. LEHMER
JOHN R. KLINE HENRY L. RIETZ

VOLUME XLI, NUMBER 11

NOVEMBER, 1935

PUBLISHED BY THE SOCIETY
MENASHA, WIS., AND NEW YORK
1935

Whole No. 434 $9.00 a Year

PUBLISHED MONTHLY

Entered as second class matter April 24, 1926 at the post office at Menasha, Wis., under the act of August 24, 1912. Acceptance for mailing at the special rate of postage provided for in the act of February 28, 1925, embodied in paragraph 4, section 538, P. L. and R., authorized May 9, 1935.

AMERICAN MATHEMATICAL SOCIETY

Mail Address: Low Memorial Library, Columbia University, New York, N. Y., for the Secretary, Treasurer, Librarian, and Clerk.

Office: Room 306, Low Memorial Library, Columbia University, New York City.

President: Professor S. Lefschetz, Princeton University, Princeton, N.J.

Secretary: Dean R. G. D. Richardson, Brown University, Providence, R.I.

Associate Secretaries: Professor M. H. Ingraham, University of Wisconsin, Madison, Wis.; Professor J. R. Kline, University of Pennsylvania, Philadelphia, Pa.; Dean T. M. Putnam, University of California, Berkeley, Calif.

Treasurer: Professor G. W. Mullins, Barnard College in Columbia University, New York City.

Librarian: Professor R. C. Archibald, Brown University, Providence, R.I.

Clerk: Miss Evelyn M. Hull, Low Memorial Library, Columbia University, New York City.

Editor-in-Chief of the Bulletin: Professor E. R. Hedrick, University of California at Los Angeles, Los Angeles, Calif.

Managing Editor of the Transactions: Professor R. D. Carmichael, University of Illinois, Urbana, Ill.

Chairman Colloquium Editorial Committee: Professor Oswald Veblen, The Institute for Advanced Study, Princeton, N.J.

Managing Editor of the American Journal of Mathematics: Professor Abraham Cohen, Johns Hopkins University, Baltimore, Md.

Representatives of the Society in the Division of Physical Sciences of the National Research Council: Professor Marston Morse, Dean R. G. D. Richardson, and Professor J. H. Van Vleck.

Representatives on the Council of the American Association for the Advancement of Science: Professors M. H. Ingraham and C. N. Moore.

Chairman, Committee on Arrangements for the Semi-Centennial: Professor T. S. Fiske, Columbia University, New York City.

THE SUMMER MEETING AT ANN ARBOR

The forty-first summer meeting and eighteenth colloquium of the Society was held at the University of Michigan, Ann Arbor, Michigan, from Tuesday to Friday, September 10–13, 1935. The Mathematical Association of America met Monday afternoon, Tuesday morning, and Tuesday evening. About two hundred seventy-five persons attended the meetings, among whom were the following one hundred seventy-seven members of the Society:

J. R. Abernethy, C. R. Adams, R. P. Agnew, N. L. Anderson, R. C. Archibald, W. L. Ayres, G. B. Banks, G. M. Bareis, W. D. Baten, Samuel Beatty, A. A. Bennett, Felix Bernstein, H. L. Black, G. A. Bliss, Henry Blumberg, J. W. Bradshaw, J. B. Brandeberry, Richard Brauer, R. W. Brink, H. E. Buchanan, J. D. Burk, W. H. Bussey, W. D. Cairns, B. H. Camp, C. C. Camp, J. W. Campbell, W. B. Carver, R. V. Churchill, C. J. Coe, L. W. Cohen, J. T. Colpitts, Max Coral, A. T. Craig, C. C. Craig, A. R. Crathorne, M. M. Culver, H. B. Curry, D. R. Curtiss, Wayne Dancer, H. T. Davis, F. F. Decker, L. L. Dines, F. G. Dressel, Ben Dushnik, P. S. Dwyer, H. P. Evans, H. S. Everett, F. J. Feinler, Peter Field, C. H. Fischer, N. C. Fisk, W. B. Ford, J. S. Frame, T. C. Fry, Lachlan Gilchrist, D. C. Gillespie, J. W. Glover, Cornelius Gouwens, L. M. Graves, E. B. Grennan, V. G. Grove, J. G. Hardy, W. L. Hart, M. L. Hartung, O. C. Hazlett, E. R. Hedrick, H. C. Hicks, T. H. Hildebrandt, J. J. L. Hinrichsen, L. A. Hopkins, Harold Hotelling, E. M. Hull, W. A. Hurwitz, M. H. Ingraham, Dunham Jackson, R. L. Jeffery, E. H. Johnson, M. M. Johnson, B. W. Jones, C. D. Jones, H. S. Kaltenborn, L. C. Karpinski, A. J. Kempner, E. E. Knight, F. W. Kokomoor, H. W. Kuhn, E. P. Lane, R. E. Langer, C. G. Latimer, V. S. Lawrence, E. R. Lorch, A. J. Lotka, C. E. Love, C. I. Lubin, E. D. McCarthy, W. H. McEwen, Roy MacKay, H. F. MacNeish, H. W. March, R. H. Marquis, W. T. Martin, R. G. Mason, L. C. Mathewson, W. O. Menge, A. D. Michal, E. W. Miller, Norman Miller, E. C. Molina, Max Morris, Richard Morris, D. C. Morrow, E. J. Moulton, J. R. Musselman, A. L. Nelson, M. M. Ness, J. A. Nyswander, F. C. Ogg, Rufus Oldenburger, E. G. Olds, F. W. Owens, H. B. Owens, T. S. Peterson, A. E. Pitcher, H. H. Pixley, L. C. Plant, V. C. Poor, Hillel Poritsky, J. E. Powell, Tibor Radó, G. Y. Rainich, W. C. Randels, J. F. Randolph, S. E. Rasor, W. T. Reid, R. G. D. Richardson, H. L. Rietz, G. de B. Robinson, S. L. Robinson, J. B. Rosenbach, H. A. Ruger, Nathan Schwid, W. A. Shewhart, C. G. Shover, T. M. Simpson, M. E. Sinclair, M. M. Slotnick, Virgil Snyder, G. G. Speeker, V. E. Spencer, C. C. Spooner, G. W. Starcher, J. D. Tamarkin, M. E. Taylor, W. C. Taylor, C. B. Tompkins, C. C. Torrance, P. L. Trump, H. S. Vandiver, J. H. Van Vleck, H E. Vaughan, C. C. Wagner, L. E. Ward, J. F. Wardwell, J. H. Weaver, K. W. Wegner, E. T. Welmers, E. A. Whitman, G. T. Whyburn, R. L. Wilder, S. S. Wilks, K. P. Williams, E. W. Wilson, L. A. Wolf, M. C. Wolf, F. E. Wood, F. L. Wren, Leo Zippin.

The colloquium lectures, on *Fermat's last theorem and related topics in number theory*, were delivered by Professor H. S. Vandiver on Tuesday, Thursday, and Friday afternoons. These lectures, for which approximately one hundred persons were registered, will appear in book form.

By invitation of the Committee on Program Professor G. Y. Rainich spoke Wednesday morning on *Remarks on product integrals and their applications to geometry*, and Professor G. T. Whyburn gave a lecture Friday morning entitled *On the structure of continua*.

A joint meeting of the Society and the newly formed Institute of Mathematical Statistics was held on Thursday morning for the completion of the organization of the Institute and for the presentation of papers on mathematical statistics. A constitution was adopted and the following officers were elected: President, Professor H. L. Rietz of the University of Iowa; Vice-President, W. A. Shewhart of the Bell Telephone Laboratories; Secretary-Treasurer, A. T. Craig of the University of Iowa. A resolution was passed that the Institute hold its meetings either in conjunction with the meetings of the American Mathematical Society or with those of the American Statistical Association.

Monday evening an informal reception was held for the visiting mathematicians at the Michigan Women's League, and Thursday afternoon a tea was given by the ladies of the Department of Mathematics of the University of Michigan. At the close of the session on Tuesday afternoon a group picture was taken. This picture will not be published, but a list of the 194 persons present may be obtained from the Secretary.

Wednesday afternoon was devoted to an excursion to Dearborn. Part of the group visited the Ford Factory, and others went to the Greenfield Village and Museum.

A joint dinner of the Mathematical Association of America, the Institute of Mathematical Statistics, and the American Mathematical Society was held Thursday evening in the ballroom of the Michigan League. Professor H. E. Buchanan acted as toastmaster. He called first upon E. H. Kraus, Dean of the College of Literature and Science, who welcomed the mathematicians to the University of Michigan. He then called upon Professor D. R. Curtiss representing the Association. Professor

H. L. Rietz described the plans of the new Institute of Mathematical Statistics, and Professor J. D. Tamarkin spoke on the unnecessary and unsatisfactory problems of an editor of a scientific journal. The dinner was attended by one hundred eighty-eight persons.

At the meeting on Friday afternoon a motion of thanks was passed to Professor Hildebrandt and the members of his department, to the President and Regents of the University of Michigan, to the management of the Ford Motor Company and the Greenfield Village, and to Professor Vandiver.

The dormitories of the University and the Women's League were open for the mathematicians and their guests.

A meeting of the Council was held at 6:00 P.M. on Tuesday in the Alumnæ Room of the Michigan League. At the business meeting held on Wednesday morning the election of the following persons to membership in the Society was announced by the Secretary:

Professor John Anderson, The Citadel, The Military College of South Carolina;
Professor Leo Avedis Aroian, Colorado State College;
Mr. Edward Thomas Ashman, Darien High School, Darien, Conn.;
Dr. Frances Ellen Baker, Vassar College;
Professor Burton Linton Beegle, Seattle Pacific College, Seattle, Wash.;
Mr. Elmer Elsworth Haskins, Northeastern University;
Professor Irving Kittell, University of Washington;
Miss Madeline Levin, Hunter College;
Professor Roy MacKay, Eastern New Mexico Junior College, Portales, N. Mex.;
Mr. Walter F. Penney, Metropolitan Life Insurance Co., 1 Madison Avenue, New York City;
Mr. Alvin Sugar, 4358 Drexel Blvd., Chicago, Ill.

As nominees of Brown University:
Mr. Nelson Dunford, Brown University;
Mr. Hugh James Hamilton, Brown University;
Mr. John Dillard Hill, Brown University;
Mr. Edward Schaumberg Quade, Brown University.

As nominee of Haverford College:
Mr. Carl B. Allendoerfer, Princeton University.

As nominees of the State University of Iowa:
Mr. Edgar Graham Harrell, State University of Iowa;
Mr. Irvin F. Keeler, State University of Iowa;
Mr. Arthur Ollivier, State University of Iowa;

Mr. Merrill Edward Shanks, State University of Iowa;
Mr. Emmet Carson Stopher, State University of Iowa.

As nominees of The Johns Hopkins University:
Dr. Edward Whitney Cannon, The Johns Hopkins University;
Dr. William Henry Erskine, The Johns Hopkins University;
Mr. R. H. Fox, The Johns Hopkins University;
Mr. James Howard Kenna, St. Mary's Seminary, Baltimore, Md.;
Mr. Richard Brandon Kershner, The Johns Hopkins University;
Mr. R. Rawhouser, The Johns Hopkins University.

As nominee of the Department of Mathematics of the University of Kentucky:
Mr. Nathan B. Allison, University of Kentucky.

As nominee of The College of St. Thomas:
Mr. Leonard William Nosker, College of St. Thomas.

As nominee of Williams College:
Professor Elmer Irwin Shepard, Williams College.

As nominee of The University of Wisconsin:
Miss Margarete C. Wolf, University of Wisconsin.

It was announced that the following person had entered the Society under the reciprocity agreement with the Deutsche Mathematiker-Vereinigung:

Dr. Olga Taussky, 39 Eckpergasse, Vienna 18, Austria.

The following thirty-one institutions of learning and mathematical groups were elected to institutional membership in the Society:

Brown University; Bryn Mawr College; University of Buffalo; University of California; University of California at Los Angeles; Case School of Applied Science; University of Cincinnati; Colorado College; Department of Mathematics of the University of Colorado; Cornell University; University of Florida; Johns Hopkins University; Department of Mathematics of the University of Kentucky; University of Michigan; New York University; Oberlin College; Ohio State University; Department of Mathematics of the University of Oklahoma; General Research Council of the State of Oregon; University of Pittsburgh; University of Rochester; Rutgers University; College of St. Thomas; Stanford University; Department of Mathematics of Swarthmore College; Syracuse University; University of Washington; Department of Mathematics of Washington University; Wells College; Western Reserve University; University of Wisconsin.

The following appointments by President Lefschetz were reported: as representative of the Society at the Seventh American Scientific Congress in Mexico City, September 8–17,

1935: Professor E. V. Huntington (Professor Huntington was also appointed official delegate of the United States Government); as additional members of the Committee on the Semi-Centennial Celebration of the Founding of the American Mathematical Society: Professors Solomon Lefschetz, M. H. Ingraham, J. R. Kline, and T. M. Putnam, all ex officio, Dr. T. C. Fry, and Professor H. S. White. (Professor R. C. Archibald was made Vice-Chairman and Dr. T. C. Fry Executive Secretary.)

Associate Secretary M. H. Ingraham presented a resumé of his year's work in the interest of the finances of the Society. A resolution of appreciation of the work done by Professor Ingraham was unanimously passed by the Council as follows:

WHEREAS, due to a change in policy of the Rockefeller Foundation which necessitated the withdrawal of their direct financial support to the American Mathematical Society, the efficient continuance of the publication program of the Society was threatened; and

WHEREAS, in order that the necessary income should be forthcoming promptly, it appeared necessary to the Council of the Society that the measures proposed for raising the additional funds made necessary by this contingency be explained in person to a large proportion of the leading universities and of the members; and

WHEREAS, Professor M. H. Ingraham was willing to undertake for the Society, with the support of the Rockefeller Foundation, this delicate and difficult task at considerable sacrifice to his personal plans; and

WHEREAS, by testimony from many sources it has become evident that his efforts have not only achieved their financial purpose, but have at the same time awakened renewed interest in and friendship for the Society among the various universities;

THEREFORE, BE IT RESOLVED: that the Council of the Society, mindful of the tactful and efficient manner in which this commission was executed, does hereby tender its profound appreciation to Professor Ingraham, and instruct the Secretary to convey to him a copy of this resolution.

This resolution was adopted by the Society, as was also a resolution of thanks to the Rockefeller Foundation in connection with their support of Professor Ingraham's undertaking.

The Secretary reported that Vice-President Vannevar Bush of the Massachusetts Institute of Technology had accepted the invitation to give the twelfth Josiah Willard Gibbs Lecture at the Annual Meeting of 1935 in St. Louis.

Professors E. P. Lane and Oystein Ore were appointed the representatives of the Society on the editorial committee of the Duke Mathematical Journal. On recommendation of the Council, the Society approved a change in the by-laws which sets aside four dollars and fifty cents of the dues of each member for a year's subscription to the Bulletin, rather than five dollars as now specified.

The following members presided at various sessions: Professors A. A. Bennett, G. A. Bliss, W. H. Bussey, D. C. Gillespie, E. R. Hedrick, T. H. Hildebrandt, Dunham Jackson, E. P. Lane, H. L. Rietz, Virgil Snyder, and R. L. Wilder. The titles and cross references to abstracts of the papers read follow: papers numbered 1 to 9 were presented Tuesday afternoon; papers numbered 10 to 13 Wednesday morning; papers 14 to 24 Thursday morning; papers 25 to 28 Thursday afternoon; papers 29 to 32 Friday morning; and papers 33 to 35 Friday afternoon. The remainder were read by title at the session on Friday afternoon. Mr. A. G. Swanson was introduced by Professor C. C. Craig, Mrs. G. T. Whyburn by Professor G. T. Whyburn, Mr. H. J. Hamilton by Professor E. R. Hedrick, and Mr. J. W. Wrench, Jr. by Professor H. M. Gehman. Paper number 7 was presented by Dr. E. R. Lorch, number 10 by Professor Virgil Snyder, number 22 by Dr. Leo Zippin, and number 26 by Professor H. E. Buchanan.

1. *Symmetric functions of matrices*, by Dr. M. C. Wolf. (Abstract No. 41-7-274.)

2. *Waring's problem and diophantine equations*, by Professor A. J. Kempner. (Abstract No. 41-9-333.)

3. *On the fundamental region of an orthogonal representation of a finite group*, by Dr. G. de B. Robinson. (Abstract No. 41-7-283.)

4. *Fundamental regions in S_{12} for the simple collineation group of order* 1092 *on seven variables*, by Professor M. M. Culver. (Abstract No. 41-9-307.)

5. *Solutions of bounded variation of the Volterra-Stieltjes integral equation*, by Dr. F. G. Dressel. (Abstract No. 41-9-324.)

6. *Generalizations of the Riemann-Lebesgue theorem.* I, by Professor R. P. Agnew. (Abstract No. 41-7-282.)

7. *The integral representation of unbounded self-adjoint transformations in Hilbert space*, by Professor F. Riesz and Dr. E. R. Lorch. (Abstract No. 41-9-323.)

8. *On Volterra-Stieltjes integral equations*, by Dr. W. C. Randels. (Abstract No. 41-9-316.)

9. *Sufficient conditions by expansion methods for the problem of Lagrange in the calculus of variations*, by Dr. W. T. Reid. (Abstract No. 41-11-379.)

10. *The Veneroni transformation in S_n*, by Professors Virgil Snyder and Evelyn Carroll-Rusk. (Abstract No. 41-9-327.)

11. *The neighborhood of a sextactic point on a plane curve*, by Professor E. P. Lane. (Abstract No. 41-7-268.)

12. *Notes on Riemannian and non-Riemannian differential geometry in abstract spaces*, by Professor A. D. Michal. (Abstract No. 41-9-303.)

13. *Variations on Bernstein's theorem*, by Professor Dunham Jackson. (Abstract No. 41-9-332.)

14. *The sampling theory of systems of variances, covariances, and intraclass covariances*, by Dr. S. S. Wilks. (Abstract No. 41-7-288.)

15. *Note on a certain bilinear form that occurs in statistics*, by Professor A. T. Craig. (Abstract No. 41-9-297.)

16. *On the frequency distributions of certain ratios*, by Professor H. L. Rietz. (Abstract No. 41-9-321.)

17. *Sheppard's corrections for a discrete variate*, by Professor C. C. Craig. (Abstract No. 41-9-299.)

18. *Factorial moments*, by Mr. A. G. Swanson. (Abstract No. 41-11-380.)

19. *Relations between two sets of variates*, by Professor Harold Hotelling. (Abstract No. 41-9-342.)

20. *An integral invariant of n-dimensional shells in $(2n-1)$-space*, by Mr. C. B. Tompkins. (Abstract No. 41-9-294.)

21. *The strong symmetrical cut-sets of closed euclidean n-space*, by Professor R. L. Wilder. (Abstract No. 41-7-267.)

22. *Periodic one-parameter groups in three-space*, by Dr. Deane Montgomery (National Research Fellow) and Dr. Leo Zippin. (Abstract No. 41-7-280.)

23. *Paratingents in topological spaces*, by Dr. C. C. Torrance. (Abstract No. 41-9-328.)

24. *Rotations around a set of fixed points*, by Mrs. G. T. Whyburn. (Abstract No. 41-11-381.)

25. *On the principles of Hamilton and Cartan*, by Professor J. W. Campbell. (Abstract No. 41-5-262.)

26. *On the characteristic exponents in certain types of problems of mechanics*, by Professors H. E. Buchanan and W. L. Duren. (Abstract No. 41-7-281.)

27. *Geometrical representation of the growth of entire functions of several complex variables*, by Dr. W. T. Martin (National Research Fellow). (Abstract No. 41-9-302.)

28. *On generalizations of length and area*, by Dr. J. F. Randolph. (Abstract No. 41-9-304.)

29. *On the fundamental number of a rational generalized quaternion algebra*, by Professor C. G. Latimer. (Abstract No. 41-7-264.)

30. *Similarity of matrices in which the elements are real quaternions*, by Dr. L. A. Wolf. (Abstract No. 41-9-331.)

31. *Characteristic spaces associated with a matrix whose elements belong to a division algebra*, by Professor M. H. Ingraham. (Abstract No. 41-9-301.)

32. *Anormal systems of numeration*, by Professor A. J. Kempner. (Abstract No. 41-9-335.)

33. *The simple group of order* 25920 (preliminary report), by Dr. J. S. Frame. (Abstract No. 41-9-293.)

34. *A classification of certain sequences of polynomials associated with persymmetric determinants*, by Miss V. E. Spencer. (Abstract No. 41-9-305.)

35. *Some theorems on the maximum sum of ordinals*, by Dr. Ben Dushnik. (Abstract No. 41-9-300.)

36. *Representation of binary forms by sets of ternary forms*, by Mr. D. M. Dribin. (Abstract No. 41-9-325-*t*.)

37. *Abstract continuous groups.* I: *Linear spaces*, by Mr. Garrett Birkhoff. (Abstract No. 41-7-266-*t*.)

38. *On the combination of topologies*, by Mr. Garrett Birkhoff. (Abstract No. 41-5-263-*t*.)

39. *Interpolation in non-regularly distributed points*, by Dr. J. H. Curtiss. (Abstract No. 41-9-308-*t*.)

40. *Properties of branch-point manifolds associated with linear*

systems of primals, by Professor T. R. Hollcroft. (Abstract No. 41-9-320-*t*.)

41. *The metric characterization of the n-dimensional hyperbolic space*, by Dr. L. M. Blumenthal (National Research Fellow). (Abstract No. 41-7-272-*t*.)

42. *Spaces of uncountably many dimensions*, by Dr. C. W. Vickery. (Abstract No. 41-9-296-*t*.)

43. *Sufficient conditions that a space be an n-dimensional manifold*, by Professor D. W. Woodard. (Abstract No. 41-9-311-*t*.)

44. *Simple algebras of degree p^e over a centrum of characteristic p*, by Professor A. A. Albert. (Abstract No. 41-7-285-*t*.)

45. *On the direct sum in circuit theory*, by Professor R. S. Burington. (Abstract No. 41-7-289-*t*.)

46. *General recursive functions of natural numbers*, by Dr. S. C. Kleene. (Abstract No. 41-7-286-*t*.)

47. *λ-definability and recursiveness*, by Dr. S. C. Kleene. (Abstract No. 41-7-287-*t*.)

48. *Formal definitions in the theory of ordinal numbers*, by Professor Alonzo Church and Dr. S. C. Kleene. (Abstract No. 41-9-319-*t*.)

49. *The probability that the mean of a second sample will differ from the mean of a first sample by less than a certain multiple of the standard deviation of the first sample*, by Professor G. A. Baker. (Abstract No. 41-7-269-*t*.)

50. *The probability that the standard deviation of a second sample will differ from the standard deviation of a first sample by less than a certain multiple of the standard deviation of the first sample*, by Professor G. A. Baker. (Abstract No. 41-7-270-*t*.)

51. *A postulational treatment of the Poisson law*, by Professors A. H. Copeland and Francis Regan. (Abstract No. 41-9-326-*t*.)

52. *A new exposition and chart for the Pearson system of frequency curves*, by Professor C. C. Craig. (Abstract No. 41-9-298-*t*.)

53. *Some arithmetic means connected with Fourier series*, by Dr. L. S. Bosanquet. (Abstract No. 41-7-271-*t*.)

54. *Transformations of multiple sequences*, by Mr. H. J. Hamilton. (Abstract No. 41-7-275-*t*.)

55. *A generalization of a cyclotomic formula*, by Dr. H. S. Grant. (Abstract No. 41-7-284-*t*.)

56. *On the Abel-Poisson summability of derived series of the*

conjugate Fourier series, by Professor A. F. Moursund. (Abstract No. 41-7-290-*t*.)

57. *On a theorem of Plessner*, by Dr. W. C. Randels. (Abstract No. 41-9-317-*t*.)

58. *Inequalities for the zeros of Legendre polynomials and related functions*, by Professor G. Szegö. (Abstract No. 41-9-318-*t*.)

59. *A note on a linear operational equation*, by Dr. F. G. Dressel. (Abstract No. 41-9-292-*t*.)

60. *Regular convergence and monotone transformations*, by Professor G. T. Whyburn. (Abstract No. 41-5-259-*t*.)

61. *Concerning rationality bases for curves*, by Professor G. T. Whyburn. (Abstract No. 41-7-291-*t*.)

62. *Sequences of transformations in a general metric space*, by Mr. J. W. Wrench, Jr. (Abstract No. 41-9-306-*t*.)

63. *Concerning distance functions and point functions*, by Dr. C. W. Vickery. (Abstract No. 41-9-295-*t*.)

64. *On extending a homeomorphism between two subsets of spheres*, by Professor H. M. Gehman. (Abstract No. 41-9-329-*t*.)

65. *Algebraic characterizations in complex differential geometry*, by Professor T. Y. Thomas. (Abstract No. 41-5-261-*t*.)

66. *Abstract euclidean spaces*. III, by Mr. I. E. Highberg, Professor A. D. Michal, and Mr. A. E. Taylor. (Abstract No. 41-9-309-*t*.)

67. *An existence theorem for a second order differential system with two-point boundary conditions in general analysis*, by Professor A. D. Michal and Mr. D. H. Hyers. (Abstract No. 41-9-310-*t*.)

68. *Note on line configurations*, by Professor W. G. Warnock. (Abstract No. 41-9-337-*t*.)

69. *Some non-involutorial space transformations associated with pencils of nodal cubic surfaces*, by Professor H. A. Davis and Dr. Amos Black. (Abstract No. 41-9-312-*t*.)

70. *A series of involutorial Cremona space transformations defined by a pencil of ruled cubic surfaces*, by Dr. Amos Black. (Abstract No. 41-9-313-*t*.)

71. *Irreducible non-projective planar graphs* (preliminary report), by Mr. I. N. Kagno. (Abstract No. 41-5-260-*t*.)

72. *Differentiable manifolds in euclidean space*, by Professor Hassler Whitney. (Abstract No. 41-9-314-*t*.)

73. *Differential geometry of a certain type of surface in S_4*, by Professor V. G. Grove. (Abstract No. 41-7-273-*t*.)

74. *On the representation of a polynomial in a Galois field as the sum of an odd number of squares*, by Professor Leonard Carlitz. (Abstract No. 41-7-277-*t*.)

75. *On sums of squares of polynomials*, by Professor Leonard Carlitz. (Abstract No. 41-7-278-*t*.)

76. *On certain higher congruences*, by Professor Leonard Carlitz. (Abstract No. 41-7-279-*t*.)

77. *Note on higher congruences*, by Professor Leonard Carlitz. (Abstract No. 41-9-336-*t*.)

78. *General relations between Bernouilli, Euler, and allied polynomials*, by Professor E. T. Bell. (Abstract No. 41-7-265-*t*.)

79. *On the matric equations $P(A,X)=0$ and $P(X)=A$*, by Professor E. T. Browne. (Abstract No. 41-7-276-*t*.)

80. *Remarks on "unsolvability,"* by Professor A. J. Kempner. (Abstract No. 41-9-334-*t*.)

81. *Binary quadratic discriminants differing by square factors*, by Professor Gordon Pall. (Abstract No. 41-9-322-*t*.)

82. *Note on the quadratic subfields of a generalized quaternion algebra*, by Professor C. G. Latimer. (Abstract No. 41-9-315-*t*.)

83. *A non-involutorial Cremona transformation belonging to the complex of secants of a twisted cubic*, by Professor H. A. Davis and Dr. Amos Black. (Abstract No. 41-9-330-*t*.)

84. *Trigonometric series and power series with repeatedly monotonic coefficients*, by Professor L. Fejér. (Abstract No. 41-9-340-*t*.)

85. *Some problems of closest approximation by means of trigonometric sums*, by Professor Dunham Jackson. (Abstract No. 41-9-341-*t*.)

86. *Fuchsian groups and ergodic theory*, by Professor Eberhard Hopf. (Abstract No. 41-11-378-*t*.)

R. G. D. RICHARDSON,
Secretary

THE TOPOLOGICAL CONGRESS IN MOSCOW

The first International Topological Conference met in Moscow, September 4–10, 1935, at the call of the Mathematical Institute of the University of Moscow. There were thirty-seven delegates, twelve from the Soviet Union and twenty-five from nine foreign countries, including ten from America; there were also about twenty visitors in attendance, mostly from parts of the USSR. The group was noteworthy for its youthfulness and vigor; in its small number it included a large proportion of the active topologists of the world. The names of the Americans present are given on page 615 of the September issue of this Bulletin. The Conference devoted all available time, except one day reserved for excursions, to papers and discussion. The papers showed a broad conception of topology and its relations with other subjects. The discussions which followed the papers were keen and informal; they revealed many instances of overlapping investigations, and led to an invaluable exchange of ideas. In fact one could hardly conceive of a more successful scientific gathering. The reception accorded the foreign delegates by the mathematicians and officials of the USSR was most cordial, and the hospitality most generous.

K. Borsuk, E. Čech, H. Freudenthal, I. I. Gordon, and S. Lefschetz presented papers on various phases of *Homology theory;* J. Nielsen, P. A. Smith, and A. Tychonoff on *Transformations;* J. W. Alexander, A. Kolmogoroff, and A. W. Tucker on *Abstract topology;* S. Cohn-Vossen, H. Hopf, and H. Whitney on *Differential manifolds;* N. K. Bruschlinsky, V. A. Efremovitch, and J. Rozanska on *Continuous representations;* C. Kuratowski, S. Mazurkiewicz, and W. Sierpinski on *Point-set theory;* Garrett Birkhoff, E. R. van Kampen, J. von Neumann, L. Pontrjagin, and A. Weil on *Continuous groups;* D. van Dantzig and M. H. Stone on *Topological algebra;* N. Bogoliuboff and V. W. Niemytzki on *Topological mechanics;* P. Heegaard on the *Four-color problem;* G. de Rham on *Lens-spaces;* A. A. Markoff on *Tresses;* O. Zariski on *Algebraic geometry;* J. Schauder on *Function-space;* W. Hurewicz on *Homotopy;* G. Nöbeling on *Triangulation;* and P. Alexandroff (president of the Conference) on *Dimension theory* and other topics.

A. W. TUCKER

THE SCIENTIFIC CONGRESS IN MEXICO

The Seventh American Scientific Congress held in Mexico City September 8–17, 1935, under the auspices of the Department of Public Education of the Republic of Mexico, was attended by over four hundred delegates, representing nearly all the countries belonging to the Pan-American Union.

Universities and scientific societies in the United States were represented by thirty delegates, of whom the following were designated as official delegates from the United States Government (without expense to the government): President C. H. Marvin, of George Washington University, chairman; President W. W. Atwood, of Clark University; Dr. J. McK. Cattell, Editor of Science; President F. S. Harris, of Brigham Young University; Professor E. V. Huntington, of Harvard University; Mr. N. M. Judd, of the Smithsonian Institution, Mr. F. V. Scholes, of the Carnegie Institution, and Mr. W. W. Schott, second secretary of the American Embassy in Mexico City. Among the universities represented were: Harvard, Yale, Chicago, California, Leland Stanford, George Washington, Clark, Brigham Young, Kansas, Miami, Minnesota, New Mexico, Pittsburgh, and the University of Washington.

The American Mathematical Society was represented by Professor E. V. Huntington. The Mathematical Association of America had appointed Professor E. L. Dodd of the University of Texas as its representative, but he was unable to attend on account of illness in his family. The American Association for the Advancement of Science was represented by Dr. J. McK. Cattell and Dr. M. S. Vallarta.

After a welcoming reception on Sunday evening, September 8, the Congress was officially opened on September 9 by General L. Cardenas, President of the Republic of Mexico, in the recently completed and imposing Palace of Fine Arts. The scientific sessions began work immediately, under the direction of Dr. P. C. Sánchez, President of the organizing committee, and Mr. L. S. Pontón, general secretary.

There were fourteen sections of the Congress, as follows: 1. Physical and Mathematical sciences; 2. Geology; 3. Engineer-

ing; 4. Industrial Chemistry; 5. Agriculture; 6. Biology; 7. Medicine; 8. Hygiene; 9. Anthropology and History; 10. Social and Economic Sciences; 11. Education; 12. Bibliography; 13. Indigenous Section; and 14. Jurisprudence. The proceedings of each section, it is understood, will be published in full by the Mexican Government.

The Section of Physical and Mathematical Sciences elected as its president Dr. J. Gallo, director of the National Astronomical Observatory at Tacubaya, Mexico, and as its secretary Dr. M. S. Vallarta, Associate Professor of Physics at the Massachusetts Institute of Technology. This section held seven sessions, at which about twenty-five papers were presented. The attendance at several of these sessions was over sixty. The papers contributed by American delegates were as follows:

E. L. Dodd, *A survey of statistical means or averages* (presented by Professor Huntington in the absence of the author).

E. V. Huntington, *An elementary determination of a rigorous equation for the figure of the earth* (illustrated with lantern slides).

M. S. Vallarta, *A theory of cosmic radiation and its experimental test.*

A resolution introduced by Dr. Gallo in favor of calendar reform on the twelve-month plan known as the World Calendar was adopted. Another resolution urged a wider adoption of the metric system in the countries of the Western Hemisphere.

In a country like Mexico, with its rich natural resources and its magnificent historical treasures, it is natural that sciences like geography and archaeology should be further advanced than the more abstract science of mathematics. But it was evident from conversations with Mexican students, many of whom have studied in the United States, that post-graduate courses in pure mathematics will before long be available in Mexico. The cooperation of American mathematicians in this plan is cordially desired. It may be remarked in passing that the social and educational problems presented by the closing of the National University of Mexico for lack of funds (which occurred while the Scientific Congress was in progress) are far too complex for any foreigner to understand or discuss.

On the social side, the Mexican authorities left nothing undone for the lavish entertainment of their guests. Of particular interest to those wishing to see the actual operation of the gov-

ernment were the excursions through the welfare colonies, model homes, and model markets, and the democratic features of the President's Reception in the National Palace on the national holiday (September 15). The all-day trips to the famous ruins at San Juan Teotihuacán, the caves at Cacahuamilpa, and the floating gardens at Xochimilco were notable examples of the generous hospitality with which the Congress was constantly surrounded. The high development of a national culture in the realm of music was evident on every hand, and the delightful and invigorating climate of Mexico City added an important element to the success of a notable gathering.

E. V. HUNTINGTON

BAKER ON GEOMETRY

Principles of Geometry. By H. F. Baker. Cambridge University Press. Volumes 1–6, 1922–1933.

Volumes 2 and 3 of this series have been reviewed separately, but there has been no review in this Bulletin of Volumes 1 and 4. Volumes 1 and 2 have been recently reprinted with slight revisions. The last two volumes were published in the latter part of 1933.

A statement by the author in the preface of Volume 5 may be taken to indicate that the series ends with Volume 6. Also, upon inquiry, the publishers have written that, so far as they know, second editions of Volumes 3 and 4 are not in preparation and the set is complete in six volumes. Mathematicians, however, will rejoice if these inferences are in error and additional volumes eventually appear. Two years ago, one might have inferred equally well that the series had ended with Volume 4!

The volumes are all neatly and durably bound in dark blue cloth. The printing and paper are excellent. The pages, however, seem over-crowded. They would be more pleasing to the reader if there were a larger number of paragraphs, more displayed equations, and if the examples, especially the longer ones in the later volumes, were in larger type.

All the volumes are indexed. In the later volumes, the indices are very good, but in the first volume, the index is far too brief to be of much use. The section titles within a chapter are pithily descriptive of the subject matter in the associated section. These section titles, with page references, are printed in full in the contents. For this reason, the contents are often of as much or more value than the index in looking up a subject reference.

The advertising leaflet issued by the publishers calls the set "a mine of geometrical lore." This describes it well. It is, indeed, a veritable gold mine, but no mine yields its treasure for nothing—one must work hard to gain it. Likewise, he who attempts merely to read these books will get little or nothing, but he who is willing to dig and delve in this mine, to amplify in scope, and complete in detail, will be repaid many times over for his time and effort by the real, but none too accessible, treasure that lies herein.

We have no quarrel with the author for maintaining the tradition that "there is no royal road to geometry." For, although the origin of this statement is traditional, the fact expressed is true of all mathematics, and all else worth while. There are certain unnecessary difficulties, however, due chiefly to arrangement. When new terms or symbols are introduced, they are often defined in obscure places to which there are usually no references in the index. The reader must scan many pages carefully to find the definitions of the symbols or terms whose meanings he can not conjecture from the context. Also, a new term is sometimes used before it is defined.

For example, in Volume 5, page 109, line 8 from top, we find, "A cubic primal in space of four dimensions contains thirty-five terms." The word "primal" is used occasionally from now on and is finally defined (p. 182),

parenthetically, in a footnote with nothing to indicate in the paragraph or sentence to which the footnote is appended that mention would be made of this term or of the idea it represents.

The new terms "prime" (Vol. 4, p. 219) and "primal" (Vol. 5, p. 182, footnote) for what have been called, none too happily, "hyperplane" and "hypersurface," respectively, evidently occurred to the author after a part of the set had been written. Both of these are timely and excellent contributions to mathematical terminology. The names "hyperplane" and "hypersurface" are unwieldy, inappropriate, and misleading—why not, as logically, "hyperline" or "hyperpoint" and "hypercurve" to designate these ideas!

Volume 1 contains a valuable bibliography. It is regrettable that this excellent feature occurs only in a modified form in Volume 2 and is entirely discontinued in succeeding volumes. In Volumes 5 and 6, however, many references are given at the end of sections and chapters.

The individual volumes will now be discussed.

Volume 1. *Foundations.* First edition, 1922. xi+183 pp. Second edition, 1929. xi+195 pp.

This volume presents the fundamentals of projective geometry, primarily in the plane and in three-space, with extensions to any number of dimensions.

Synthetic methods are used chiefly in this and in most of the succeeding volumes. The author's point of view as to method is well expressed in the following excerpt from the preface: "The geometrical theory is accompanied by an algebraic symbolism, which serves to help to fix ideas, and for purposes of verification; it is necessary also to include the proof that this symbolism is appropriate to the purpose. It is held, however, that the geometrical argument should be complete in itself, independently of the symbols; a geometry should have such a comprehensive grasp of geometrical relations that all its results are clear by consideration of the geometrical entities alone."

The following changes occur in the second edition:

The corrections given in Volume 2, first edition, page 238, and in Volume 3, page 224, are made in the text.

Three notes, on pages 184–194, are added, amplifying certain stated portions of this volume. In each of the lists at the end of chapters 2 and 3 occurs one additional problem. The only erratum (p. viii) refers to the second of these problems.

The excellent bibliography, on pages 176–183, is revised and slightly enlarged.

Volume 2. *Plane Geometry. Conics, Circles, Non-Euclidean Geometry.* First edition, 1922. xv+243 pp.* Second edition, 1930, xv+259 pp.

The following changes occur in the second edition:

The corrections given on page 224, Volume 3 are made in the text.

The bibliography of Notes 2 and 3 is slightly enlarged.

More extended discussions (Additions, pp. 239–253) of certain problems and theorems of the text account for the additional pages of the second edition. In the body of the book, these are referred to by the abbreviation "Add." appended to the problem or theorem to be further considered.

---

* Reviewed by F. S. Woods, this Bulletin, vol. 31 (1925), pp. 370–371.

Volume 3. *Solid Geometry. Quadrics, Cubic Curves in Space, Cubic Surfaces.* 1923. xix+228 pp.*

Corrections to this volume are given in Volume 4, p. 244.

Volume 4. *Higher Geometry. Being illustrations of the utility of the consideration of higher space, especially of four and five dimensions.* 1925. xvi+250 pp.

The author states in the preface that the fourth volume was written first and that the other three are chiefly introductory to it.

In this volume, the last three chapters are the most important. Chapters 1 and 3 are introductory to these. Chapters 2 and 4, however, are inserted because of their own interest and value and are not necessary for succeeding chapters. For this reason, these two chapters will be briefly treated before taking up the main sequence.

Chapter 2 is devoted to the discussion of the Hart circle, a circle determined by any three given circles such that four other circles touch this circle and each of the original three. All eight circles are in one plane.

Chapter 4 deals with the configuration in S_4 corresponding to the set of twenty-seven lines on the cubic surface in S_3.

In Chapter 1, certain basic relations are treated, first in two and three, then in three and four, and finally in five dimensions. At first glance, the arrangement of topics seems haphazard, but evidently they have been very carefully chosen by the author in carrying out his purpose to give, not a complete discussion, but a sufficient number of selected illustrations to enable the interested reader to fill in and complete the picture himself.

Chapters 3 and 6 are so closely related that they are best treated together. In Chapter 3 are developed the properties of the elliptic plane quartic, obtained by projection of the intersection quartic of two quadrics of S_3. This is done in considerable detail to serve as a basis for Chapter 6 in which the cyclide, a special quartic surface of S_3, is studied as the projection of the intersection surface of two quadric primals of S_4. Dupin's cyclide and other special forms are included in the discussion.

Chapter 5 takes up the study of the configuration of fifteen lines and fifteen points in S_4 (see frontispiece) and the associated loci, many of which are old friends introduced in a new way.

Chapter 7 deals chiefly with relations in five dimensions associated with Kummer's quartic surface.

The treatment throughout is largely synthetic. In general, topics are developed by consecutive examples, which are stated and either are solved or have directions given for their solutions. Often the connecting discussion between adjacent examples must be supplied by the reader.

This volume covers a wide range of material, chosen with a purpose and accomplishing that purpose—to demonstrate the well nigh indispensable utility of higher space in the derivation and study of properties of configurations in ordinary space.

Volume 5. *Analytical Principles of the Theory of Curves.* 1933. x+247 pp.

The last two volumes, which appeared almost simultaneously, are extremely rich in variety and scope. In the preface of Volume 5, the author tells why these two volumes were written.

* Reviewed by B. H. Brown, this Bulletin, vol. 32 (1926), pp. 173–174.

"In the last fifty years, a remarkable advance has been made in the theory of surfaces, and of algebraic loci in general; the English reader may find a description of the nature of this in a Presidential Address to the London Mathematical Society given* in November, 1912. But attempts, since the War, to expound these new results have continually shown the necessity for a precise appreciation of the ideas out of which this advance has developed; in mathematics it is not sufficient to know the enunciation of a result; it is necessary to understand the proof. These two volumes have grown up in the attempt to meet this need."

Volumes 5 and 6 differ from the preceding volumes in that algebra is freely used and the discussion follows the more usual form with examples employed chiefly for illustration or extension.

The author assumes that the reader has a knowledge of the projective properties of higher plane curves. There has been no adequate treatment, in fact, scarcely any treatment of this subject, not even of plane cubics and quartics (except the elliptic quartic) in the preceding volumes. This, however, is in line with the author's statement that he does not attempt to complete any subject and with his evident aim not only to impart information, but to stimulate the reader to do as much of the necessary work himself as possible.

In the first chapter, birationally equivalent manifolds are defined. Rational and elliptic plane curves are discussed as the two simplest examples of such equivalence. In Chapter 2, the linear series on a curve, defined in Chapter 1, is used to reduce a curve with given singularities to a non-singular curve.

For the next five chapters, some familiarity with the theory of functions is essential. The algebraic functions for a curve and Abel's Theorem for the sum of algebraic integrals are discussed. The integral genus is defined and is shown to be identical with the arithmetical genus. The linear series on a curve is now treated in more detail with applications and examples in the four notes (pp. 86–110). Chapters 5 and 6 deal with algebraic integrals, especially in regard to their periods and the relation of these to Riemann surfaces.

In Chapter 7, the modular forms of rational functions and integrals are introduced and used to derive anew some of the results of the preceding chapters.

In the first part of the final chapter, the relations among the properties of curves of S_3 are derived by projective methods and certain relations are shown to have interpretations in the light of Chapter 4. The relations for space curves which are complete or partial intersections of two surfaces are now obtained and the linear series on a space curve studied. The chapter closes with a few applications to curves of higher space.

Volume 6. *Introduction to the Theory of Algebraic Surfaces and Higher Loci.* 1933. ix+308 pp.

In this volume, the first three chapters deal with the theory of correspondence and the last four with surfaces in three and four dimensions.

In Chapter 1, point correspondences on the same curve or on two curves are the chief topics of the first part; part 2 introduces transcendental methods

* Proceedings of the London Mathematical Society, vol. 12 (1913), pp. 1–40.

and part 3, defective integrals, with applications to the theory of correspondence.

In the second chapter, the theory of correspondence is extended to points and primes in higher space. The treatment employs the method and notation of Schubert's symbolic calculus.

Chapter 3 deals briefly with transformations and involutions of the plane, including a short resume of Bertini's four types. Many references to the extensive literature on this subject are given in the course of the discussion.

In Chapter 4, relations of projective properties of surfaces in three and four dimensions are treated, with numerous examples and references. On page 157, the author introduces a new term, "accidental double points," for the term, "improper double points," used by Severi in the paper cited in the footnote on this page. An excellent reason is given (p. 158) for Severi's choice of a name for these points, but no grounds are offered for the rejection of this name and the substitution of another. Moreover, the use of the adjective, "accidental," in this connection seems inappropriate, since all surfaces of S_4 which are projections of surfaces of S_5 (with the exception of cones and the Veronese quartic) have improper double points, that is, such a point on a surface in S_4 is no more accidental than the double curve of a surface in S_3 or the double point of a plane curve.

Chapter 5 deals with the theory of invariants of birational transformations of a surface in three dimensions. In the preface, the author states that this chapter "is concerned with the most interesting and the most novel ideas of the volume." The discussion is intended to be only introductory and many references are given for those who wish to continue it further.

In Chapter 6, the intersections of loci in S_4, especially primals, are treated in some detail.

The last chapter contains particular theorems and examples, and also applications of some of the theory developed in the preceding chapters.

This series is the most comprehensive treatise on geometry that has ever appeared in English. To be sure, there are many gaps, such as the inadequate treatment of higher plane curves, but these omissions consist chiefly of routine matter, amply covered in several current treatises in English and other languages. Volumes 4, 5, and 6 contain the principal contributions of the series.

This work is also remarkable because of its emphasis on the synthetic method. The attitude of the author that geometry should, of itself, be able to justify its conclusions reminds one of Steiner. The author, however, evidently does not "hate" analysis, as Steiner is said to have done, but uses it with energy and finesse whenever he deems it preferable.

Mathematicians, especially geometers, are deeply indebted to Professor Baker for his invigorating presentation of this basic branch of our science.

T. R. HOLLCROFT

SHORTER NOTICES

Integrationsmethoden der Lieschen Theorie. By Dr. Gerhard Kowalewski. (Mathematik und ihre Anwendungen, Volume 15.) Leipzig, Akademische Verlagsgesellschaft, 1933. viii + 221 pp.

The author restricts himself to a relatively small number of topics, which he treats with considerable detail and illustrates with numerous well-selected exercises. He gives the same brilliant treatment and employs the same stimulating style evidenced in his earlier work: *Einführung in die Theorie der kontinuirlichen Gruppen*, Volume 9 of the same series. (For a review of this book, see this Bulletin, vol. 40 (1934), p. 20.)

There are only two chapters. In the first and much the longer one, the author considers ordinary differential equations invariant under r-parameter groups and methods for solving such equations. He develops the theory of invariants of r-parameter groups and brings out their use in the problem of solving differential equations invariant under such groups. He emphasizes clearly the importance of Lie's determinant Δ, and later extends the use of this important tool by introducing the *mixed* Lie determinant. Numerous well-chosen illustrative exercises aid one greatly in following the theory. He does not confine himself to Lie's treatment of these important topics, but gives an account of his own work and that of his students, either in detail or by reference to the literature.

In the second chapter the author confines himself to the study and use of Lie's integrating factor (first announced by Lie in 1874) for an ordinary differential equation, invariant under a one-parameter group, and to an extension of this idea to systems of ordinary differential equations of the first order.

The book is decidedly a valuable addition to the available literature on the topics treated.

ABRAHAM COHEN

Application des Imaginaires au Calcul Vectoriel. By J. B. Pomey. Paris, Gauthier-Villars, 1933. vi+70 pp.

The subtitle is "Complément à la Conférence sur le Calcul Vectoriel." This course, given at the École Supérieure d'Électricité, deals in part with the use of complex numbers in vector analysis. The book under review furnishes material for this part of the course. The author gives various applications of complex numbers to geometry, kinematics, alternating currents, elliptic vibrations, and the method of symmetric coordinates of Fortescue. There is also a discussion of Cayley's formula, which sets up a correspondence between the bilinear transformation of the complex plane and the rotation of a sphere about a diameter. The book is carefully written and the treatment in several places is somewhat unusual. It makes stimulating reading for the mathematician and gives some notion of the kind of mathematics used in electrical engineering.

C. A. SHOOK

La Méthode de Repère Mobile, La Théorie des Groupes Continus, et Les Espaces Généralisés. By Elie Cartan. (Actualités Scientifiques et Industrielles, No. 194.) Paris, Hermann, 1935. 65 pp.

In this booklet based on a series of lectures delivered in Moscow in 1930, Cartan develops some of the ideas of modern differential geometry from the point of view of ennuples of vectors. He begins with an examination of the method of the moving trihedral in classical differential geometry, observing that underlying this method are the two following conditions: 1) The trihedral, defined at each point of the manifold under consideration, must be determined intrinsically by the differential elements of the manifold, and 2) corresponding trihedrals of congruent curves (or surfaces) must be congruent. Several problems of classical differential geometry—minimal curves, homothetic curves, plane curves equivalent under the group of unimodular affine transformations, surfaces whose two fundamental forms have one linear factor in common—are considered in the light of his analysis of the method of the moving trihedral, and by the application of this method such topics as generalizations of the Frenet-Serret equations, curvature, arc, for these manifolds are developed.

The reader is thus led very naturally and easily to the more general problem—the application of these ideas to the fundamental principles of the theory of continuous groups. The way is quickly opened for a number of general theorems, Lie's constants of structure, and the Darboux-Maurer-Cartan equations of structure. The author investigates the role of these equations in a number of particular cases, and observes that *"les équations de structure du groupe G contiennent en elles toute la géométrie différentielle de l'espace doué du groupe fondamental G, ..."* The last chapter is devoted to a brief, yet valuable, presentation of a number of questions in the theory of surfaces, Riemannian spaces, ruled spaces, and the more general linearly connected spaces.

This booklet, profusely illustrated with relatively simple examples, should be extremely valuable to all who wish to gain some insight into the meaning of the mass of generalizations that have been developed in this field in recent years. Unlike other works in this field, this one does not lose the reader in a maze of computations; yet the ideas are exhibited clearly and strikingly. The method of treatment employed serves to make this booklet unusually readable, while the emphasis on the geometric aspects not only adds to the lucidity, but serves to remind that modern differential geometry is geometry and not merely a branch of tensor analysis and differential equations.

HARRY LEVY

Intermediate Mechanics; Dynamics. By D. Humphrey. London, Longmans Green, 1930. xi+382 pp.

This is a good elementary text on analytical mechanics in the English tradition, offering a more explicit treatment of the elementary parts but not going as far as such standard texts as Jeans, Lamb, or Loney. Seven of the nine chapters are devoted to kinematics and dynamics of a particle in one or two dimensions, including the inevitable chapters on projectiles and simple harmonic motion; but two pages are given to motion under a central force, the reader being referred to more advanced books and advised that "the most important case is

when F is a gravitational force varying as the inverse square"! The book closes with two chapters on the most fundamental points in the dynamics of rigid bodies. The book might prove of value in a class of unusually gifted freshmen requiring more than the usual first term college mechanics, or for sophomores, but is, in the opinion of the reviewer, hardly advanced enough for upper class majors in physics or mathematics.

H. P. ROBERTSON

La Causalité des Théories Mathématiques. By G. Bouligand. (Actualités Scientifiques et Industrielles, No. 184.) Paris, Hermann, 1934. 41 pp.

This booklet essays an investigation of the nature of generalizations by analysing the ultimate "causes" of mathematical proofs. The author distinguishes sharply between *algorithmic* proofs, which depend upon calculations, and *direct* proofs, which closely follow the intuition and so explain the real causes of the theorem. For example, the direct methods in the Calculus of Variations avoid artificial assumptions of differentiability. The same holds of the so-called "direct infinitesimal geometry"; in particular, Lebesgue's surface applicable to the plane but containing no line segment whatever is a striking revelation of the misleading results of algorithmic calculations.

A group of transformations can be viewed as a domain of causality. By this the author apparently means that the hypothesis and the conclusion of a geometric theorem should be invariant under the same group; if this is not the case the hypothesis or the conclusion can be generalized to give a more extended theorem. The numerous examples given include extensions of Meusnier's theorem and of the theorem of Dupin on triple orthogonal systems. This investigation of causes leads the author to argue for the objectivity of mathematics—that mathematics is a science studying a "mathematical nature" (the meaning of this naïve phrase is not explained). The objective causes in this science are obtained not only by the discovery of proofs, but also in the formulation of axiom systems and in the discovery of significant concepts (for instance, the notion of semi-continuity). Such an emphasis on the importance of definitions is indeed refreshing.

SAUNDERS MACLANE

Matière et Énergie. By Victor Henri. Paris, Hermann, 1933. 436 pp.

This book is a solid and well-written text in modern physics. The introductory chapters take up the kinetic atomic theory of matter, x-ray spectra and atomic number, and the periodic system of the elements. These are followed by an account of natural and artificial radioactivity and the role of isotopes in the periodic system, with emphasis on the experimental conditions and findings; these chapters offer of course but a beginning in nuclear physics and must be supplemented by reports on later work. The last 200 pages present the elementary parts of kinetic theory and the Bohr theory of atomic structure, again with emphasis on the empirical findings. The book should be found very useful in obtaining a descriptive orientation in the complex fields touched upon, preparatory to a more detailed and more mathematical study; for such a purpose it can be highly recommended.

H. P. ROBERTSON

Wave Mechanics; Advanced General Theory. By J. Frenkel. Oxford, Clarendon Press, 1934. viii+525 pp.

This second part of a general treatise on wave mechanics is primarily devoted to the "mathematical development of the general ideas underlying the new mechanics." The following are the headings of the nine chapters: *Classical mechanics as the limiting form of wave mechanics*; *Operators*; *Matrices*; *Transformation theory*; *Perturbation theory*; *Relativistic remodelling and magnetic generalization of the wave mechanics of a single particle*; *The problem of many particles*; *Reduction of the problem of a system of identical particles to that of a single particle*; *Second (intensity) quantization and quantum electrodynamics.* The name of the author is sufficient guarantee of the interest of the book to students of wave mechanics and the work reflects credit upon both the author and publisher. It is inevitable, however, that a comparison be made between the present work and the treatises of Dirac and von Neumann. In such a comparison it is at once evident that Frenkel relies much more than do Dirac and von Neumann upon analogy, and it is the present reviewer's opinion that this reliance is unfortunate. While the analogy between relativistic wave mechanics and Maxwell's electromagnetic theory of light, discussed at length in the book, is interesting, in the end an argument from analogy is helpless before the cant question of the day "and so what?" The book gives a good account of the Pauli theory and of Dirac's four-dimensional matrix theory, and ends with Breit's formula improving the quantum electrodynamics of Heisenberg, Pauli, and Dirac. It is significant that the last paragraph of the book begins as follows: "Now it seems quite certain that this new theory is in principle just as wrong as the old one. . . ."

F. D. MURNAGHAN

Signification de l'Histoire de la Pensée Scientifique. By F. Enriques. (Actualités Scientifiques et Industrielles, No. 161.) Paris, Hermann, 1934. 68 pp.

That scientific concepts can be understood only in terms of their historical development is the thesis of this pamphlet. The author has given a persuasive and vigorous presentation of this "genetic" philosophy of science and of the consequent importance of the history of science. He infers that science is not a body of absolute doctrine but a series of approximations to the truth; that there is a "unity" of all the sciences; that there is a continuity in scientific discovery. The latter point is applied to the background of the Newton-Leibniz controversy. Many other suggestive remarks are made, for example: the genetic analysis of abstract concepts runs counter to the axiomatic approach.

SAUNDERS MACLANE

NOTES

In addition to the members of the editorial staff of the Bulletin, the following persons have assisted the editors, either by refereeing papers or by advising concerning papers offered for publication in the present volume: C. R. Adams, Harry Bateman, E. T. Bell, B. A. Bernstein, H. R. Brahana, Daniel Buchanan, J. R. Carson, C. Chevally, E. W. Chittenden, A. B. Coble, L. E. Dickson, E. L. Dodd, G. C. Evans, Phillip Franklin, Raymond Garver, L. M. Graves, Einar Hille, T. R. Hollcroft, E. V. Huntington, W. A. Hurwitz, M. H. Ingraham, R. D. James, A. J. Kempner, W. S. Kinne, E. P. Lane, R. E. Langer, N. H. McCoy, C. C. MacDuffee, W. A. Manning, H. W. March, A. D. Michal, H. H. Mitchell, C. N. Moore, F. D. Murnaghan, Oystein Ore, W. F. Osgood, G. Y. Rainich, J. F. Ritt, J. A. Shohat, P. A. Smith, Virgil Snyder, M. H. Stone, E. B. Stouffer, D. J. Struik, J. D. Tamarkin, J. M. Thomas, Stepan Timoshenko, A. W. Tucker, J. V. Uspensky, H. S. Vandiver, J. L. Walsh, H. S. White, Hassler Whitney, G. T. Whyburn, W. M. Whyburn, Norbert Wiener, A. R. Williams, W. A. Wilson, B. C. Wong. The editors desire publicly to acknowledge this service.

The School of Mathematics of the Institute for Advanced study each year allocates a small number of stipends to gifted young mathematicians and mathematical physicists to enable them to study and to do research work at Princeton. Candidates must have given evidence of ability in research comparable at least with that expected for the degree of Doctor of Philosophy. Blanks for application may be obtained from the Institute, and are returnable by February 1, 1936.

Part I of E. H. Moore's *General Analysis* has just been published in the Memoirs of the American Philosophical Society. It is a book of vi+251 pages devoted to the algebra of matrices in a form especially adapted to general analysis. The three parts to follow are entitled: *The fundamental notions of general analysis, Generalized Fourier series and modular spaces,* and *The characteristic value problem of general analysis.*

Professor V. Volterra, formerly of the University of Rome, has ceased to belong to the Accademia dei Lincei. His place as national member in the section of mechanics has been taken by the corresponding member General G. A. Crocco, professor at the School of Aeronautical Engineering in Rome. Professor U. Amaldi, of Rome, and Professor L. Berzolari, of Pavia, have been named national members in the section of mathematics. The following have been elected corresponding members: C. Bompiani, of Rome; A. Comessatti, of Padua; M. De Franchis, of Palermo; B. Levi, of Bologna; A. Signorini, of Naples.

The Italian Society of Sciences (the XL) has awarded the prize of mathematics for the period 1929–1933 to Professor V. Bernstein, of Pavia, for his researches in the theory of analytic functions and especially Dirichlet's series.

The Johnson Memorial Prize of the University of Oxford for 1935 has been awarded to Dr. T. G. Cowling, Brasenose College, lecturer in mathematics at the University College of Swansea, for an essay entitled *Convection in stars.*

Dr. W. N. Bailey, senior lecturer in mathematics at the University of Manchester, has been awarded a Leverhulme Research Fellowship. The subject of his researches will be *The study of functions of hypergeometric type.*

Professor Felix Bernstein has been awarded a research fellowship at Columbia University.

Mr. H. M. MacNeille, of Harvard University, has been awarded a Sterling Fellowship at Yale University.

Dr. Lars Ahlfors, of the University of Helsingfors, will be a visiting lecturer at Harvard University this year.

The retirement of Professor Max Dehn, of the University of Frankfurt a. M., is announced.

Professor B. Finzi, of the University of Milan, and Professor F. Ibrana, of the University of Genoa, have been promoted to professorships in rational mechanics.

The retirement is announced of Professors Heinrich Leibmann and Anton Rosenthal, of the University of Heidelberg.

Dr. Wilhelm Maier, formerly of the University of Frankfurt, for the last three years a member of the department of mathematics at Purdue University, has resigned to join the faculty of the University of Freiburg. Dr. Maier has been spending the summer in Germany.

Dr. Karl Mayrhofer has been promoted to a professorship in mathematics at the University of Vienna.

Dr. Jerzy Neyman, lecturer in mathematical statistics at the Central College of Agriculture, Warsaw, has been appointed reader in statistics at University College, London.

The title of professor of statistics has been conferred on Dr. E. S. Pearson by the University of London, in honor of the post held by him at University College.

Professor B. Segre has been promoted to a professorship in projective and descriptive geometry at the University of Bologna.

Professor Otto Szász is visiting a series of eastern universities during the first semester of this academic year on a grant from the Institute of International Education.

The note regarding the Seventh American Scientific Congress on page 615 of this volume was incorrect in stating that Professor M. A. Basaco was a representative of the American Mathematical Society at this Congress.

The note regarding Professor Samuel Beatty on page 481 of this volume of this Bulletin was in error: he has held a professorship for many years, and

was appointed head of the department of mathematics at the University of Toronto in 1934.

Professor Richard Brauer, of the Institute for Advanced Study, has been appointed to an assistant professorship at the University of Toronto.

Dr. A. B. Brown, of Columbia University, has been promoted to an assistant professorship.

Assistant Professor W. A. Conrad, of the United States Naval Academy, has been promoted to an associate professorship.

Associate Professor Tobias Dantzig has been promoted to a professorship in mathematics and has been appointed executive officer in the department of mathematics at the University of Maryland.

Dr. L. A. Dye, of Cornell University, has been appointed assistant professor of mathematics at The Citadel (Charleston, S. C.).

Dr. C. C. Grove, of the College of the City of New York, has been promoted to an assistant professorship.

Mr. E. H. Hanson has been appointed director of the department of mathematics and professor of mathematics at the North Texas Teachers College.

Assistant Professor L. M. Kells, of the United States Naval Academy, has been promoted to an associate professorship.

Mr. W. F. Kern, of the United States Naval Academy, has been promoted to an assistant professorship.

Dr. G. B. Lang, of the University of Illinois, has been appointed professor of science and mathematics at West Georgia College.

Assistant Professor Otto Laporte, of the University of Michigan, has been promoted to an associate professorship in physics.

Dr. Hans Lewy, of Brown University, has been appointed a lecturer at the University of California.

Mr. G. A. Lyle, of the United States Naval Academy, has been promoted to an assistant professorship.

Professor E. J. McShane, of Princeton University, has been appointed professor of mathematics at the University of Virginia.

Professor C. C. MacDuffee, of Ohio State University, has been appointed to a professorship in mathematics at the University of Wisconsin.

Assistant Professor W. O. Menge, of the University of Michigan, has been promoted to an associate professorship in mathematics.

Dr. Deane Montgomery, of Harvard University, has been appointed an assistant professor of mathematics at Smith College.

Dr. M. E. Mullings has been appointed head of the department of mathematics at Abilene Christian College, Texas.

Dr. P. K. Rees has been appointed to an assistant professorship at State College, New Mexico.

Associate Professor J. B. Rosenbach, of the Carnegie Institute of Technology, has been promoted to a professorship in mathematics.

Professor G. T. Sellew, of Knox College, has been named professor emeritus of mathematics.

Dr. G. C. Webber, National Research Fellow, is working at the University of Pennsylvania this academic year. Dr. Webber was at Brown University last year.

Professor G. T. Whyburn, of the University of Virginia, has been appointed chairman of the department of mathematics there.

Associate Professor R. L. Wilder, of the University of Michigan, has been promoted to a professorship in mathematics.

Associate Professor L. T. Wilson, of the United States Naval Academy, has been promoted to a professorship.

The following appointments to instructorships in mathematics are announced: Agricultural and Mechanical College of Texas: Dr. E. C. Klipple; University of Alabama: Dr. P. M. Hummel; Antioch College: Mr. Max Astrachan; University of Arizona: Dr. E. J. Purcell; Brown University: Dr. B. C. Getchell, Mr. H. E. Vaughan; College of the City of New York (Evening Session): Dr. D. S. Nathan; University of Florida: Mr. E. S. Quade; Hood College: Dr. Ruth G. Mason; Johns Hopkins University: Dr. J. H. Curtiss; Kent State College (Kent, Ohio): Dr. Frances Harshbarger; University of Maryland: Mr. C. B. Tompkins; Massachusetts Institute of Technology: Dr. R. H. Cameron; Mt. Holyoke College: Dr. F. E. Baker; University of Nebraska: Dr. A. A. Stafford, Mr. M. S. Webster; University of Nevada: Mr. H. C. Ayres; North Carolina State College of Agriculture and Engineering: Dr. Jack Levine; Ohio State University: Dr. C. E. Rhodes, Dr. C. R. Wylie, Jr.; Princeton University: Dr. E. W. Titt; Purdue University: Dr. A. H. Smith; University of Rochester: Dr. Wladimir Seidel; University of Texas: Mr. O. H. Hamilton; United States Naval Academy: Dr. N. R. Ball, Dr. Randolph Church, Dr. A. E. Currier, Mr. J. R. Hammond, Dr. S. B. Littauer, Dr. T. W. Moore; University of Wisconsin: Dr. S. C. Kleene; Xavier University: Mr. J. F. Butler.

Harvard University expects to appoint two Benjamin Peirce Instructors in mathematics for the academic year 1936–37. These instructorships are intended to give promising young men the opportunity to combine their research activities with a moderate amount of teaching, including some advanced instruction. Applications should be sent to Professor W. C. Graustein, Chairman of the Division of Mathematics, by February 15, 1936.

The death of Professor A. von Brill, of the University of Tübingen, is reported.

Professor G. G. Chambers, of the University of Pennsylvania, died on October 31, 1935, from a thrombosis condition.

Associate Professor Raymond Garver, of the University of California at Los Angeles, died after a brief illness on Thursday, November 7, 1935, at the age of thirty-four.

Professor D. C. Gillespie, of Cornell University, died suddenly on October 31, 1935, at the age of fifty-seven.

Professor E. H. Jones, of the Southern Methodist University, Texas, died on August 7, 1935.

Dr. C. E. Mendenhall, professor of physics and chairman of the department of physics at the University of Wisconsin, died on August 18, 1935, at the age of sixty-three years.

Professor W. M. Thornton, professor emeritus of applied mathematics, formerly chairman of the faculty at the University of Virginia, died on September 11, 1935, at the age of eighty-four years.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

356. Mr. Garrett Birkhoff: *The composition of modular algebras.*

By a *modular algebra* is meant a group, with or without operators, ring, or hypercomplex algebra. The paper is a systematic study of the ways of building up complex modular algebras from simple constituents. The main contributions are: (1) the general formulation of some new methods of combining algebras, (2) the exact correlation of analogous methods of combination, (3) the extension and simplification of some results of Remak (Journal für Mathematik, 1930–1932) concerning canonical decompositions of finite groups. (Received October 1, 1935.)

357. Mr. Nelson Dunford: *A particular sequence of step functions.*

It is the purpose of this note to establish the existence of a sequence of positive step functions $f_n(t)$ having the property that $f_n(t)=0$ except for a set whose measure approaches zero with $1/n$ and such that for every summable function $g(t)$, except for those in a certain set of the first category in L, the sequence $\int_0^x f_n(t)g(t)dt$ is everywhere dense in the space of measurable functions. (Received September 30, 1935.)

358. Mr. Nelson Dunford: *Linear operations on L_p to L_q.*

It is the purpose of this paper to establish the following representation theorems. The general linear operation on L to L_2 is given by the formula $Tf=\int_0^1 K(P, Q)f(P)dP$ where the kernel $K(P, Q)$ is measurable, is in L_2 for each P, and the ess.$_P$sup. $\left[\int_0^1 |K(P, Q)|^2 dQ\right]^{1/2}<\infty$. This bound gives the norm of T. For a measurable kernel satisfying the relation $\left[\int |K(P, Q)|^q dQ\right]^{1/q} \in L_{p/p-1}$ the above transformation is completely continuous on L_p to L_q and the general linear operation is always the limit of a sequence of transformations of this type. The general linear operation on L to a Hilbert space X is given by $Tf=\int x(P)f(P)dP$ where $x(P)$ is an essentially bounded and measurable function with values in X and the integration is in Bochner's sense. Every abstract measurable function $f(P)$ on (0, 1) to L_q and satisfying the condition $\|f(P)\| \in L_p$ can be represented by a measurable function $K(P, Q)$ such

that $\int[\int|K(P, Q)|^q dQ]^{p/q} dP < \infty$ and conversely. (Received September 30, 1935.)

359. Dr. E. J. Finan: *A theorem on matrices.*

The purpose of this article is to prove the following result: THEOREM. *Let $A=(a_{rs})$ be a square matrix of order n with elements in a finite algebraic field $\Gamma(\theta)$ where Γ is the rational field. Let a_{tt} satisfy an equation $f(\lambda)=0$ of lowest degree m in Γ. Now form the direct product $I \cdot x\ A$ which is composed of the n^2 scalar matrices $[a_{rs}, a_{rs}, \cdots, a_{rs}]$. Replace the scalar matrix in the tt position by the companion matrix C of $f(\lambda)=0$. The characteristic function $\phi(\lambda)$ of the resulting matrix P contains the characteristic function of A as a factor. In fact $\phi(\lambda)\equiv\prod_{i=0}^{m-1} f_i(\lambda)$ where $f_i(\lambda)$ is the characteristic function of A with a_{tt} replaced by its ith conjugate and $f_0(\lambda)=f(\lambda)$.* (Received September 18, 1935.)

360. Professor Tomlinson Fort: *Maxima and minima of finite sums.*

In this paper the author gives a general discussion of the maxima and minima of finite sums with the development of a new sufficient condition in terms of the solutions of a recurrent relation. Several particular problems are treated including the development of a necessary form for the minimum surface of revolution obtained by revolving a broken straight line about an axis. (Received September 21, 1935.)

361. Professor Philip Franklin: *Classification of measurable functions.*

The author begins with a series of theorems, for the most part well known, on the possibility of approximating Riemann integrable, summable, and measurable functions by continuous functions. These lead to a classification of functions with some advantages over a similar one due to Carathéodory. The author concludes with some results on B. a. p. functions equivalent in the sense of Besicovitch. (Received September 30, 1935.)

362. Mr. Michael Goldberg: *A class of multi-symmetric polyhedra.*

This note shows the construction of all the multi-symmetric trihedral convex polyhedra possessing only hexagons in addition to twelve pentagons (or six quadrilaterals or four triangles) regularly and symmetrically distributed. Among these are found sets of polyhedra which have the anomalous property of being topologically different even though they possess the same number of polygons in their sea of hexagons and their twelve pentagon islands (or six quadrilaterals or four triangles) are regularly and symmetrically disposed. (Received September 13, 1935.)

363. Professor Harold Hotelling and Margaret Richards Pabst: *Rank correlation and tests of significance involving no assumptions of normality.*

The rank correlation coefficient is treated primarily as a test of the existence

of relationship between two variates, rather than as a measure of correlation. In this way it is possible to find exact probabilities without making any assumption of a bivariate normal population. A proof is given for the first time that the distribution approaches normality of form for large samples, a proposition generally assumed for practical purposes, which, however, is apparently not a corollary to any of the known general theorems on approach to normality. Other tests of significance that avoid the assumption of an underlying normal distribution are discussed. A method of combining the evidence for association contained in a 2 by s contingency table, using both χ^2 and the order relations of the frequencies, is illustrated. (Received September 5, 1935.)

364. Professor Edward Kasner: *Relative conformal invariants.*

A fundamental problem of conformal geometry is to find the invariants of a horn angle (two curves having ordinary contact at a given point). The author, in 1910, proved the existence of a unique absolute invariant, and gave its geometive interpretation in the form $I_3=(d\gamma_1/ds_1-d\gamma_2/ds_2)/(\gamma_1-\gamma_2)^2$, where γ_1 and γ_2 are the curvatures of the given curves (see Kasner, Proceedings of the International Congress of Mathematicians, Cambridge, 1912, p. 81). From the proof it is obvious that the denominator $(\gamma_1-\gamma_2)^2$ is a relative invariant of second order, and likewise the numerator $d\gamma_1/ds_1-d\gamma_2/ds_2$ is a relative invariant of third order. In a recent paper Ostrowski proves again the relative invariance of $\gamma_1-\gamma_2$, which he denotes by the symbol $\beta(1, 2)$, and he gives many beautiful applications of this simple invariant (Jahresbericht der Deutschen Mathematiker-Vereinigung, 1934). In the present paper it is shown that each of these relative invariants is by itself sufficient to characterize the conformal group. The absolute invariant I_3 serves to characterize conformal among all contact transformations. If three curves are in contact, then $(\gamma_1-\gamma_2)/(\gamma_1-\gamma_3)$ is not only an absolute conformal invariant, but also an absolute topological (analytic restricted) invariant. An absolute topological invariant Ω_6 involving the curvatures of six curves was given by the writer in the American Journal of Mathematics, 1906, p. 210, with application to the result for the conformal case on p. 213. The equilong group gives rise to an absolute univariant J_3 which is the dual of I_3. In particular the dual of Ostrowski's relative invariant is $\gamma_1-\gamma_2$, the difference of the radii of curvatures. (Received September 14, 1935.)

365. Professor Edward Kasner and Dr. George Comenetz: *Conformal geometry of horn angles.*

A horn angle is the figure formed by two curves which have (simple) contact at a point. Under the group of conformal transformations in the plane, a horn angle has a single absolute invariant of finite order; this is the fraction $(\delta_1-\delta_2)/(\gamma_1-\gamma_2)^2$, where γ_1, γ_2 are the curvatures of the two curves at the point, and δ_1, δ_2 are the rates of change of curvature with respect to arc-length (Kasner, *Conformal geometry*, Proceedings of the Cambridge Mathematical Congress (1913), vol. 2, p. 81). If n curves are drawn through a point, all in the same direction, they form $C_{n,2}$ horn angles. The $C_{n,2}$ values of the invariant are not all independent when n exceeds 3. It is found, in fact, that if four curves are drawn, forming six horn angles, the six values of the invariant must satisfy

a certain long equation of twelfth degree, which is given explicitly. (Received September 27, 1935.)

366. Professor Hans Rademacher: *On prime numbers on real quadratic fields in rectangles.*

The existence of a totally positive fundamental unit gives rise to a periodicity in certain finite sums, of which the number of prime numbers in a rectangle is a special case. The periodicity is utilized for a Fourier expansion, which leads to an identity for those finite sums. In the case of prime numbers some estimates can be obtained, one of which has been already given by the author in Acta Arithmetica, vol. 1 (1935). (Received September 28, 1935.)

367. Professor J. F. Ritt: *On the singular solutions of algebraic differential equations.*

This paper undertakes the study of the singular solutions of algebraic differential equations, particularly of equations of the second order, from the point of view of the theory of irreducible manifolds. (Received September 27, 1935.)

368. Dr. M. S. Robertson (National Research Fellow): *On the coefficients of an odd schlicht function.*

If $f(z)=z+\sum_{n=1}^{\infty}b_{2n+1}z^{2n+1}$ is regular and schlicht for $|z|<1$ it is known that $|b_{2n+1}|<A$ for all n, where A is an absolute constant. In particular $|b_3|\leqq 1$, $|b_5|\leqq e^{-2/3}+1/2$. When the coefficients are all real, then $|b_{2n-1}|+|b_{2n+1}|\leqq 2$. In this paper the author demonstrates that the latter inequality is also true for $n=1$, $n=2$, when the coefficients are complex, numbers, and that $(|b_3|+|b_5|)/2\leqq[(|b_3|^2+|b_5|^2)/2]^{1/2}\leqq 1$. (Received September 28, 1935.)

369. Dr. M. S. Robertson (National Research Fellow): *On the univalency of the Cesàro partial sums of a univalent function.*

If $f(z)=z+\sum_{2}^{\infty}a_nz^n$ is regular and univalent for $|z|<1$, the partial sums $s_n(z)=z+\sum_{k=2}^{n}a_kz^k$ are in general not univalent in the unit circle, but are univalent for $|z|<1/4$ at least. Under certain conditions, as J. W. Alexander has shown, the partial sums are, however, univalent for $|z|<1$. The author shows that whenever all the partial sums $s_n(z)$ are univalent in $|z|<1$ so also are univalent for $|z|<1$ all the Cesàro sums of the first order. (Received September 28, 1935.)

370. Mr. L. B. Robinson: *A system of Riquier and the tensor calculus.*

Given a complete system $X_if\equiv A_{i1}\partial f/\partial x_2+A_{i2}\partial f/\partial x_3+\cdots+A_{ir}\partial f/\partial x_{r+1}=0$, $(i=1, 2, \cdots, n)$, where A_{ik} depends on $x_1, x_2, \cdots, x_k$ only, is rational with respect to x_1, and is rational and integral with respect to the other variables, by quadratures one can solve such a system in finite form. Let it be called a system of class K. The tensors of a system of linear homogeneous differential equations satisfy the following system of Riquier: $\partial u_i/\partial x_1=U_{i1}+\sum_{j=1}^{r}S_{1j}\partial u_i/\partial S_j, \partial u_i/\partial x_2=U_{i2}+\sum_{j=1}^{r}S_{2j}\partial u_i/\partial S_j, \cdots, \partial u_i/\partial x_n=U_{in}+\sum_{j=1}^{r}S_{nj}\partial u_i/\partial S_j, (i=1, 2, \cdots, n)$, (see Riquier, *Les Systèmes d' Équations aux Dérivées*

Partielles, p. 502). The integration of the above system can be reduced to the integration of a system of class K. The author has obtained some similar results in the field of functional equations. (Received September 18, 1935.)

371. Professor J. A. Shohat: *On mechanical quadratures.*

First, the general mechanical quadratures formula based on the Lagrange interpolation formula is discussed, with regard to the signs of its "Cotes' numbers" and the distribution of the abscissas employed. Thus some of the results of Stekloff are obtained and extended. Second, the special mechanical quadratures formula is discussed, which uses for abscissas the zeros of the polynomial $\phi_n(x)+A\phi_{n-1}(x)+B\phi_{n-2}(x)$, where A, B are constants and $\{\phi_n\}$ are orthogonal Tchebycheff polynomials. The discussion leads to more precise statement and also an extension of recent results of Fejér (Mathematische Zeitschrift, vol. 37 (1933), pp. 287–309). (Received September 25, 1935.)

372. Dr. Abraham Sinkov: *On the simple group of order* 5616.

In 1931 Dr. K. E. Bisshopp obtained necessary and sufficient conditions which two operators, S and T, of periods three and two must satisfy in order to generate the simple group G_{5616}. The theorem consisted of two abstract definitions of the group, each of which required nine distinct restrictions upon the generators. Apparently no attempt was made to study the independence of these relations. The purpose of the present paper is to simplify the results given by Bisshopp and to show that five conditions are sufficient to define the group completely in either case. Four of these conditions give the periods of the generators, their product and their commutator: (3, 2, 13; 4) in one case and (3, 2, 13; 6) in the other. (Received September 10, 1935.)

373. Miss V. E. Spencer: *Note on asymptotic expressions for the zeros of Hermite and Laguerre polynomials.*

By applying the relation between Hermite and the generalized Laguerre polynomials to a result of Zernike asymptotic expressions for the largest zero x_{nn} of Laguerre polynomials with $\alpha=1/2$ and $3/2$ are immediately obtained accurate to a term in $n^{-5/2}$. By use of Markoff's theorem and a certain argument similar to Zernike's the theory is extended to asymptotic expressions for x_{nn} for any α. (Received September 28, 1935.)

374. Dr. W. R. Thompson: *On confidence ranges for the median and other expectation distributions for populations of unknown distribution form.*

Let $A=\{x\}$ denote the real number domain; and U, called an *infinite population*, denote an unknown frequency-distribution law of draft from $\{x\}$ such that the probability of obtaining x in an arbitrary interval, $P_{(\alpha<x<\beta)}=\int_\alpha^\beta f(x)\cdot dx$, is unknown, but $f(x)\geqq 0$ and bounded in A; and, for every positive $p<1$, there exists (α, β) such that $P_{(\alpha<x<\beta)}<p$. Let n drafts, independently thus governed, be made from A without replacements, yielding a sample denoted by $\{x_i\}$ enumerated so that $i<j$ for $x_i<x_j$, $i, j=1, \cdots, n$. Then it is shown that the well known principle of Bayes is strictly applicable to evalua-

tion of several expectations, $\overline{P}_{(R)}$, where R is a relation between sample values and population parameters; for example, if we let p_k denote $P_{(x<x_k)}$, then $\overline{P}_{(p_k<p)}=I_{p(k,n-k+1)}$, the incomplete B-function ratio; if M is the unknown population median of U, then $\overline{P}_{(x_k<M<x_{k+1})}=(1/2)^n\cdot\binom{n}{k}$; and $\overline{P}_{(x<x_k)}=k/(n+1)$. Similar relations are developed for a finite population, and best upper and lower bounds are found for some bivariate cases of expectation. (Received September 26, 1935.)

375. Dr. S. E. Warschawski: *On the preservation of angles at a boundary point in conformal mapping.*

Let R be a simply connected "schlicht" region in the w-plane whose boundary contains $w=0$. Let $w=0$ be "accessible" along the Jordan curve L. Suppose that there is a circle $|w|<\rho$ such that the part of the boundary of R inside this circle lies in the angles: (*) $|\arg w-h_+|\leqq k_+$, $|\arg w-h_-|\leqq k_-$, $(h_-\leqq h_+)$. Suppose, furthermore, that L connects $w=0$ with a boundary point outside $|w|=\rho$ such that L divides R into two subregions. Let all boundary points of one subregion, which are in $|w|<\rho$ and not on L, be in one of the angles (*), and those of the other subregion, which are in $|w|<\rho$ and not on L, be in the other. Then the author proves: If $w(z)$ maps $|z-1|<1$ conformally on R such that $z=0$ "corresponds" to the boundary point $w=0$ defined by L, then $|\arg w(z)-H(\arg z)|\leqq K(\arg z)+\epsilon$, where $\epsilon\to 0$ as $z\to 0$ in any angle $|\arg z|\leqq\theta<\pi/2$, and $H(\alpha)=(1/\pi)[\{(\pi/2)+\alpha\}h_++\{(\pi/2)-\alpha\}h_-]$, $K(\alpha)=(1/\pi)[\{(\pi/2)+\alpha\}k_++\{(\pi/2)-\alpha\}k_-]$. This result is an extension of a theorem of Carathéodory and Lindelöf. (Received September 27, 1935.)

376. Professor Hassler Whitney: *On a theorem of H. Hopf.*

Let K be a complex. Let the "inverse boundary" of an i-cell E^i be the sum of the (properly oriented) $(i+1)$-cells which have E^i on their boundaries. The author defines in the obvious manner the inverse boundary of any i-chain, inverse cycles, inversely bounding cycles, and hence inverse Betti groups $\overline{B}^i$ (compare Alexander, Proceedings of the National Academy of Sciences, August, 1935; also Whitney, ibid., July, 1935). If f is a map of the n-complex K^n onto the n-sphere S^n, it may be deformed so that all $(n-1)$-cells lie on a point of S^n; each cell E_k^n then lies on S^n with a definite degree γ_k. Now $\sum\gamma_kE_k^n$ is an inverse cycle, determining an element of $\overline{B}^n$. A simple proof is given that the classes of maps of K^n onto S^n are in one-one correspondence with the elements of $\overline{B}^n$. It is not hard to show that this is equivalent to a theorem of Hopf (Commentarii Mathematici Helvetici, vol. 5 (1933)). (Received September 3, 1935.)

377. Dr. Max Zorn: *Elimination of the continuum-hypothesis from algebra.*

Given a system of elements with a definition of dependence satisfying van der Waerden's axioms (see *Moderne Algebra*, p. 204), the author shows that two bases are not only similar but may also be well-ordered such that the sections of elements of equal indices without immediate predecessors are equivalent. Weakening the axioms such that they cover rational dependence the author proves the existence of bases and, in the infinite case, the similarity of all

bases. This similarity theorem yields Steinitz's degree of transcendency, and Baer's degree of infinite algebraic extensions. The axiom of choice is used, but well-ordered is involved only by some of its cardinal consequences. The generalized continuum-hypothesis, essential in Baer's theory, is completely avoided. (Received September 27, 1935.)

378. Professor Eberhard Hopf: *Fuchsian groups and ergodic theory.*

This paper deals with geodesics on surfaces of constant negative curvature. Every geodesic is supposed to be continuable indefinitely in both directions. It is shown that the flow connected with the geodesics problem is always metrically transitive if the surface area is finite. A special case has been treated previously by Hedlund. The method developed in the present paper is novel. The principal tools are potential theory and the *NE* metric introduced by Poincaré. In the case of infinite surface area it is shown that geodesics tend, in general, towards the funnels of the surface. (Received September 10, 1935.)

379. Dr. W. T. Reid: *Sufficient conditions by expansion methods for the problem of Lagrange in the calculus of variations.*

In the present paper a sufficiency theorem for a strong relative minimum in the problem of Lagrange is established by the expansion method used by Levi (Annali di Matematica, vol. 21 (1913), pp. 173–218) for the simple problem of the calculus of variations in the plane. The hypotheses of the theorem do not involve any assumption of normality, and have the same generality as those used by Hestenes (Transactions of this Society, vol. 36 (1934), pp. 793–818) in the proof of sufficiency theorems by the usual field method for the general problem of Bolza. Use is made of the result relating the strengthened conjugate point condition and the solutions of the accessory differential equations which has been established independently by Morse (Transactions of this Society, vol. 37 (1935), pp. 147–160) and the author (American Journal of Mathematics, vol. 57 (1935), pp. 573–586). The extension of Levi's method to the problem of Lagrange is not trivial. In particular, it involves a proof of a certain inequality which is satisfied by the Weierstrass $\mathfrak{E}$-function for large values of certain of its arguments. The expansion method may also be extended to prove the results established by Hestenes for the general problem of Bolza. (Received September 10, 1935.)

380. Mr. A. G. Swanson: *Factorial moments.*

Using the generating function of factorial moments, as given by Campbell, as a definition, a formula in the form of a derivative evaluated at a point is obtained for finding the frequency function of a variable which may take on a finite set of values and for which the factorial-moment-generating-function is known. This formula is made use of in obtaining the distribution of means in samples of N independent observations of a variable with known factorial-moment-generating-function. A generalization of a theorem of Pólya is established giving conditions under which if a sequence of factorial-moment-generating-functions has as its limit an ordinary moment generating function, the

corresponding frequency functions has as its limit the function corresponding to the limiting moment generating function. Two methods are used in finding the characteristics of the distribution of $\sigma_{(n)}$ (the nth factorial moment) in samples of N. First a system of symmetrical functions analogous to the semi-invariants of Thiele are introduced and by means of their properties a method of computation of the factorial moments of $\sigma_{(n)}$ is obtained. The second method is based on the known method of finding semi-invariants of moments about a fixed point in samples of N. The first four factorial moments of the first four factorial moments are given. (Received September 12, 1935.)

381. Mrs. G. T. Whyburn: *Rotations around a set of fixed points.*

Let $T(M)=M$ be a topological transformation of a closed set of points M into itself, and let K denote the set of fixed points under T. It is shown first that the components of $M-K$ break up into cyclic rotation groups of finite or infinite order in such a way that the boundary of any one element of a given group is identical with the boundary of any other element of that group. Furthermore, if M is a plane continuous curve, the boundary of any element of a group of order >2 contains at most two points. If M is a topological sphere, there can exist only one rotation group of order >1. Furthermore this is the only rotation group and it is exactly of order 2. (Received September 12, 1935.)

382. Dr. F. G. Dressel: *A note on Young-Stieltjes integrals.*

The paper presents an expansion formula for $\int_a^b f(x)dg(x)h(x)$, serviceable even though $g(x)$ and $h(x)$ have common points of discontinuity. (Received October 3, 1935.)

383. Dr. Saunders MacLane: *The Schönemann-Eisenstein irreducibility criteria in terms of prime ideals.*

By using non-archimedean absolute values in the construction of the ideal factors of a rational prime it is possible to obtain a general simplified statement of the irreducibility theorems of Eisenstein and Schönemann. This statement, which includes practically all the successive generalizations of these theorems for polynomials with rational coefficients (Bauer, Dumas, Kürschák, Ore, Rella, MacLane), runs as follows: Consider the polynomial $f(x)$ as if it were an irreducible equation defining an algebraic field, and use Newton polygons (or absolute values) to construct the prime ideal decomposition of a prime p in this purported algebraic field. If p then appears to be a power p^e of a prime ideal P of apparent degree f, where $e \cdot f$ is the degree of $f(x)$, it follows that the original polynomial $f(x)$ is actually irreducible. (Received October 5, 1935).

384. Dr. Saunders MacLane: *A construction of prime ideals in terms of absolute values of an algebraic field.*

The problem under consideration is the construction of all the prime ideal factors of a rational prime p in an algebraic field $R(\theta)$ defined by a given equation $f(x)=0$. Ore's convenient method for this construction in terms of Newton

polygons is known to fail in certain exceptional cases. These exceptions can be avoided by a general and abstract formulation of the procedure in terms of absolute values. This depends on the known fact that every prime ideal of $R(\theta)$ corresponds to a non-archimedean absolute value of $R(\theta)$. But each such absolute value is equivalent to an absolute value of the polynomial ring $R[x]$, in which $f(x)$ and all its multiples have the value $+\infty$. These values of $R[x]$ are in turn constructed by an extension of the method outlined in a previous paper by the author (Proceedings of the National Academy, vol. 21 (1935), p. 472). The arithmetic properties of the discriminant of $f(x)$ show that this method of construction will give all the prime ideal factors of p after a finite number of steps. The results (including the degrees and exponents of the prime ideals) can be formulated without recourse to ideal-theory. (Received October 5, 1935.)

385. Dr. Solomon Kullback: *On certain distributions derived from the multinomial distribution.*

The multinomial distribution gives the probabilities for the various combinations of events possible in N trials where in each trial one of n mutually exclusive events occur. In the N trials some of the possible events may not occur, others may occur once, twice, . . . , each. The general problem we consider is that of studying the simultaneous distribution of the number of events which occur $r, s, \cdots, t$ times each. The theory is illustrated and verified by specific examples. The theory discussed has certain practical applications. (Received November 2, 1935.)

386. Professor C. C. Latimer: *On the class number of a quaternion algebra with a negative fundamental number.*

Let A be a rational generalized quaternion algebra with a negative fundamental number, i.e., containing an element with a negative norm. It is shown that every one-sided ideal in an arbitrarily chosen set G' of integral elements in A is a principal ideal. A special set G of integral elements which contains a certain ring G_1 is considered. It is known that there is a correspondence between the classes of regular left ideals in G_1 and certain classes of Hermitian forms. From this it is shown that every left ideal in G is a principal ideal. The first result mentioned above follows from this and a result of Brandt's. Theorems on the existence of a g.c.r.d. and a g.c.l.d. of two elements and on the factorization of an element in G' are obtained. These results are similar to well known results of Hurwitz for a certain set in the classic quaternion algebra. (Received October 11, 1935.)

387. Professor L. A. Dye: *Involutorial space transformations associated with a rational ruled surface.* II.

Two involutorial transformations are studied in this paper which arise from a one-one correspondence between the cubic surfaces of a pencil and the generators of a ruled quartic surface. Each surface of a pencil of cubic surfaces having two fixed double points is tangent to a plane along the line l through the nodes. This plane cuts a line r from the surface and these lines r generate a

ruled quartic surface having l as a triple line. A point P of space selects a cubic surface and hence a line r. The plane Pr is tangent to the quartic surface at a point Q on r. The line PQ meets the quartic surface in a residual point P', the associate of P in an involutorial transformation. A similar transformation arises when a pencil of cubic surfaces with a double line is used. In each of these transformations the generators of the quartic surface are parasitic lines of the transformation and the quartic surface is shed off from the transformation three times. (Received October 14, 1935.)

388. Professor Arnold Emch: *Two notable classes of hypersurfaces.*

Since the Cayley cubic and the Steiner quartic surfaces are dual to each other and have other interesting properties, they have always attracted considerable attention among geometricians. In this paper two dual classes of hypersurfaces in S_{2n-1} are discussed which reflect in a generalized form many of the properties of the Cayley and Steiner surfaces. The author calls these Cayley and Steiner hypersurfaces. (Received October 16, 1935.)

389. Professor R. F. Rinehart: *Some properties of the discriminant matrices of a linear associative algebra.*

MacDuffee and L. E. Bush investigated some of the properties of the discriminant matrices of a linear associative algebra by reducing them to a diagonal form by means of transformations of basis of the algebra. Thence they derived a set of cyclic relations among the constants of multiplication, by means of which it is possible to derive some interesting properties of the discriminant matrices and the characteristic equations. In the present paper a revision of the reduction of Bush (this Bulletin, vol. 38 (1932), pp. 49–51) is given and consequences of the cyclic relations are further developed, with particular attention to the case of a non-simple algebra. The implications of the equality of the discriminant matrices are considered, and sufficient conditions for their equality are derived. Applications of some of these results to the particular case of the algebra generated by an algebraic equation are noted, yielding, in particular, new viewpoints on the Newtonian identities and the Borchardt-Jacobi theorem. (Received October 21, 1935.)

390. Dr. Leo Zippin: *Note on a problem of E. Čech.*

Professor E. Čech has introduced the following definition of local connectedness: A space shall be said to be locally connected provided every finite covering by open sets contains a finite covering by connected sets. He has raised the question whether such a space (given that it is a regular topologic space) is necessarily bicompact. The author proves the following theorem: *If a compact topologic space (not necessarily regular) is locally connected in the usual sense, then it is locally connected in the sense of Čech.* This leads easily to a negative solution of the problem. However, for regular spaces, the property of Čech does imply *compactness*, and also local connectedness in the usual sense. (Received October 23, 1935.)

391. Dr. Leo Zippin: *On monotonic complete covering systems.*

Professor Lefschetz has raised the question of how far the projection-sequence theory of Alexandroff, which has its origins in compact metric spaces, can be carried over (admitting *uncountable* "sequences") to the homology theory of Čech, which embraces abstract spaces. The author proves a theorem which has the following consequence: in order that a Hausdorff space possess a *monotonic complete* system of finite coverings by open sets, it is necessary and sufficient that it be compact metric. In this direction, then, no generalization seems possible. (Received October 23, 1935.)

392. Dr. Solomon Kullback: *On certain factorial sums.*

The problem here considered may be stated as follows: Evaluate the sum $S_{rs\cdots t}(n, N, a_1, a_2, \cdots, a_n)=\sum [N!/(x_1!x_2!\cdots x_n!)] (a_1^{x_1}a_2^{x_2}\cdots a_n^{x_n})$ where $a_1, a_2, \cdots, a_n$ are any constants and the summation is for all values of $x_1, x_2, \cdots, x_n$, such that $x_1+x_2+\cdots+x_n=N$ with no x equal to $r, s, \cdots, t$. Only positive integral values (including zero) of $x_1, x_2, \cdots, x_n, N, r, s, \cdots, t$ are considered. (Received October 24, 1935.)

393. Professor J. J. Stoker, Jr.: *The form of surfaces of positive curvature in three-dimensional space.*

The starting point of this paper is a theorem due to Hadamard (Journal de Mathématiques Pures et Appliquées, (5), vol. 3 (1897)) which is usually stated in the following form: A closed surface in three-dimensional space which is free of singularities and whose Gaussian curvature is everywhere positive is topologically equivalent to the sphere, that is, it is an ovaloid. The purpose of the present paper is to generalize this theorem for open surfaces. At the same time another proof of the theorem of Hadamard is given. An essential assumption in proving the theorems of this paper is that of the *completeness* of the surfaces. It is shown that every *open* surface which is complete possesses the following properties: (1) it is topologically equivalent to the plane, (2) it is the boundary of an unbounded convex point set, (3) the spherical representation of the surface lies on a hemisphere and the correspondence set up between the points of the surface and those of the spherical representation is one-to-one, hence the surface possesses a total Gaussian curvature $\leqq 2\pi$, (4) the *entire* surface admits of a representation in the form $z=f(x, y)$ with f one-valued. (Received October 24, 1935.)

394. Professor C. N. Moore: *On necessary and sufficient conditions for convergence factors in multiple series.*

In a paper presented at the February meeting of the Society in New York (see this Bulletin, abstract 41-1-56) the author gave sufficient conditions for convergence factors in series summable by Nörlund means. The object of the present paper is to modify those conditions in such a manner as to make them both necessary and sufficient. The results thus obtained constitute a generalization to multiple series of the theorems for single series contained in a note recently published in the Proceedings of the National Academy of Sciences, vol. 21 (1935), pp. 263–266. (Received October 25, 1935.)

395. Professor C. C. MacDuffee: *A recursion formula for the polynomial solutions of differential equations with constant coefficients.*

Spampinato (Atti della Reale Accademia Nazionale dei Lincei, Rendiconti (6), vol. 21 (1935), pp. 73–76) has used the companion matrix of the auxiliary equation of a homogeneous differential equation with constant coefficients to obtain infinitely many solutions. The method is here extended to obtain a recursion formula for all homogeneous polynomial solutions of each degree. Computation by this formula is rapid. (Received October 28, 1935.)

396. Professor W. D. Baten: *Frequency distributions of means of n independent variables whose frequency distributions can be dissected into component frequency laws.*

This paper develops distributions of arithmetic averages of n independent variables under the assumption that the frequency distribution of each of the variables can be dissected into two component frequency distributions. A general situation is first presented after which special cases are considered. The last section shows that the Gaussian law is equal to the sum of an infinite number of Gaussian laws. (Received October 29, 1935.)

397. Professor L. E. Dickson: *A new method for Waring theorems with polynomial summands.* II.

Instantaneous deduction of a Waring theorem for a polynomial $f(x)$ of degree $2n$ from a known Waring theorem for a polynomial of degree n is described. In a recent number of the Transactions of this Society the case in which $f(x)$ involves only even powers of x was treated. Here the new case in which $f(x)$ contains also a term in x is discussed. (Received October 30, 1935.)

398. Dr. C. W. Vickery: *Abstract vector spaces of uncountably many dimensions.*

The author has obtained, for each ordinal number α, a set $\mathfrak{G}_\alpha$ of axioms in terms of the undefined notions, *vector*, $+$, $\cdot$, $\| \ \|$, χ, and Σ, such that if S is any set of objects called vectors satisfying these axioms, then S is isomorphic and isometric with the set of all vectors of space D_{ω_α}. In this treatment no topological notions are employed (overtly). The notions of an uncountable basis and the sum of an uncountable sequence of vectors are introduced. (Spaces D_{ω_α} are defined in a paper recently presented to the Society.) (Received October 30, 1935.)

399. Professor L. W. Cohen: *Cauchy convergence in topological space.* Preliminary report.

The notions of Cauchy convergence, total boundedness, and completeness are defined in topological spaces satisfying the first denumerability axiom. It is proved that if the space is complete every totally bounded set is compact. A sufficient condition for the converse is given. The problem of immersing such spaces topologically in complete spaces is discussed. (Received October 31, 1935.)

400. Mr. H. Reingold: *Generalized determinants of Vandermonde.* II.

Let G_{ij}, $(i, j=1, 2, \cdots, n)$, be a square matrix of g, let $G_{rs}^{(m)}$ stand for the element in the rth row and the sth column of the mth power of the matrix G_{ij}, and let $G_{ij}^{(0)}$ denote the Kronecker δ. Consider the n by n determinant $|G_{r_i s_1}^{(m_1)} G_{r_2 s_2}^{(m_2)} \cdots G_{r_n s_n}^{(m_n)}|$, $(i=1, 2, \cdots, n)$, in which the ith row is exhibited, where $r_1, r_2, \cdots, r_n$ and $s_1, s_2, \cdots, s_n$ are any specific permutations of the integers $1, 2, \cdots, n$, and $m_1<m_2<\cdots<m_n$. Denote this determinant by $\Delta(m_1, m_2, \cdots, m_n)$. In a previous paper (see this Bulletin, abstract 41-5-230) certain properties of these determinants were considered. In the present note a more general theorem is stated of which the previous results are special instances. This last property may be stated in the following form: $\Delta(m_1, m_2, \cdots, m_n)=g\cdot\Delta(m_1-1, m_2-1, \cdots, m_n-1)=g^{m_1}\cdot\Delta(0, m_2-m_1, \cdots, m_n-m_1)$. In case the matrix G_{ij} is symmetric, the last result of this paper may be obtained by a method supplied by Professor G. Szegö, namely, that of reducing the associated quadratic form to a sum of squares. (Received October 31, 1935.)

401. Professor F. John: *On the moments of inertia of convex regions.* I.

A convex region in the plane may be called "normal," if the ellipse of inertia corresponding to its center of gravity is a circle. If d denotes the greatest, Δ the least diameter, and A the area of a normal region R, then $d\leqq 2^{1/2}\Delta$, $A\leqq\Delta^2$. If R is symmetrical with respect to a point, then also $A\geqq d^2/2$. Moreover, if S is a convex body, then through an interior point of S there passes a plane such that its intersection with S is normal. (Received November 1, 1935.)

402. Professor F. John: *On the moments of inertia of convex regions.* II.

Let R be a normal convex region with a center O, (i.e., R may be symmetrical with respect to a point O, and the ellipse of inertia of R corresponding to O may be a circle). If a and b are the sides of any rectangle circumscribed about R, then $a/b\leqq\alpha$, where $\alpha=3[2+(100)^{1/2}]^{-1/2}=1.164\cdots$ (α being the exact constant in this connection). (Received November 1, 1935.)

403. Mr. C. B. Tompkins: *A note on complete covariant differentiation.*

E. H. Cutler (Transactions of this Society, vol. 33 (1929), p. 832) has defined by example complete covariant differentiation. It is a method of adding quantities to the derivatives of a complete tensor to make a complete tensor. A set of quantities, $A_{\alpha\beta\cdots\phi}^{ab\cdots g}$ whose indices may have several ranges forms a complete tensor if it transforms as a tensor under transformations of coordinates to which each of its index ranges appertains. In this paper complete covariant differentiation is considered from the standpoint of the coefficients of connection. To do this a Douglas space of paths is set up; it leads to unsurprising conclusions. This space has a number of dimensions equal to the sum of the numbers of dimensions appertaining to the various types of indices encountered. It

may be possible to reduce the number of dimensions of this introduced space. This problem is equivalent to finding conditions for the tensor analysis of a space to be isomorphic with the tensor analysis of a subspace and a variety in that subspace. These conditions are similar to generalized Gauss and Codazzi equations of the variety. (Received November 1, 1935.)

404. Professor W. J. Trjitzinsky: *Theory of linear differential equations containing a parameter.*

An equation of nth order is considered with coefficients indefinitely differentiable, in x, on a certain interval. These coefficients are given asymptotic to series in negative integral powers of a complex parameter λ, with a finite number of positive powers of λ possibly present. Important results, under various restrictions, were previously obtained by G. D. Birkhoff, J. D. Tamarkin, and P. Noaillon. In the present work *the fundamental existence theorem establishes the asymptotic character of the solutions under no restrictions on the formal series solutions.* This theorem is applied to several problems. The paper will appear in the Acta Mathematica. (Received November 1, 1935.)

405. Professor Tobias Dantzig: *Note on the history of elliptic functions.*

One of the earliest contributions to the history of elliptic integrals was made by di Fagnano in 1716. The idea was later generalized by Euler, and eventually, at the hands of Legendre and Jacobi, led to the addition theorems. Let $F(x) = \int_0^x [(Ax^2+B)/(Cx^2+D)]^{1/2}dx$: Fagnano considers the relation $F(x)+F(\bar{x}) = k \cdot \bar{x}$, and shows that for certain values of the constant k this generally transcendental expression reduces to a biquadratic in x and $\bar{x}$. If $F(x)$ be regarded as the length of an arc in a central conic, the relation establishes an algebraic correspondence between two points of the conic. In the present paper, there is given a geometrical interpretation of the Fagnano correspondences. It turns out that of the two cases studied by Fagnano, one reduces to $R\bar{R}=ab$, and the other to $y\bar{y}=b^2/a$, where a and b denote the semi-axes of the conic, and y, $\bar{y}$, and R, $\bar{R}$ are the ordinates and the radii of curvature at corresponding points. (Received November 1, 1935.)

406. Professor L. E. Dickson: *On Waring's problem and its generalization.*

By a modification of the latest method of Vinogradow in the current Annals of Mathematics, his small number of nth powers whose sum yields every large integer N is reduced by ten or more. The same conclusion is obtained when the nth powers are multiplied by any given positive integer. A C is found for which the above results hold for every $N>C$. Universal Waring theorems are then deduced. If q denotes the greatest integer $<(3/2)^n$, then $q^{2^n}-1$ is the sum of $I=q+2^n-2$, but no fewer, nth powers. No integer has been found which requires more than I powers. This ideal I is 4223, 8384, 16673 when $n=12, 13, 14$. When $n=12$ to 17 it is proved that every positive integer is a sum of I nth powers. Moreover, for $n=12$ or 13, every integer $\geq 2\cdot 3^n$ is a sum of 2757 or 4342 nth powers; every integer $\geq 4^{14}$ is a sum of 5238 fourteenth powers. These results fail for slightly smaller integers. (Received October 15, 1935.)

407. Professor Hassler Whitney: *A function not constant on a connected set of critical points.*

A function $f(x, y)$ with continuous first derivatives is defined such that $\partial f/\partial x = \partial f/\partial y = 0$ at every point of a nonrectifiable arc, while the function is not constant on the arc. The paper will appear in the Duke Mathematical Journal. (Received October 25, 1935.)

408. Dr. E. L. Post: *Polyadic groups.* Preliminary report.

In 1928 W. Dörnte generalized the concept of abstract group to allow for an operation involving an arbitrary number of elements m. In the present investigation the concepts of substitution and transformation are also generalized. An identity is seen to be an $(m-1)$-ad of elements, and corresponding definitions of inverse and transform result. The author shows that every abstract m-adic group (Dörnte's m-group) can be represented as an m-adic substitution group. There results the following theorem. Every abstract polyadic group can be represented as a coset of an invariant subgroup of an ordinary group (and conversely for finite groups). Further work is largely restricted to finite groups and includes, for abstract groups, a complete generalization of the theory of cyclic groups and of the first and third parts of Sylow's theorem under the condition g/p^B prime to $(m-1)$. For substitution groups results on odd and even substitutions, sets of intransitivity, and Jordan's theorem for regular groups have been generalized, while for linear groups the elementary theorems, other than for similarity transformations, go over. Still in progress is the determination of the m-adic linear groups in two variables for the case $m=3$. (Received October 26, 1935.)

409. Professor Louis Weisner: *Criteria for the compositeness of finite groups.*

This paper includes a number of compositeness criteria which are closely related to those of Frobenius and Burnside. Let G be a group of order $p^\alpha m$, where p is a prime that does not divide m, let H be a subgroup of order p^α of G, and let $N(P)$ denote the normalizer in G of a subgroup P of G. The following are among the criteria given. 1. A necessary and sufficient condition that G have an invariant subgroup of index p that does not include a particular element S of H is that H have a maximal subgroup I that does not include S, such that every conjugate of S under G that is contained in H have the form SV, where V is an element of I, while every conjugate of $S_p{}^e (e=1, \cdots)$ under G that is contained in H is contained in I. 2. If H is an abelian group, a necessary and sufficient condition that G have an invariant subgroup of index p is that $N(H)$ have an invariant subgroup of index p. 3. If a subgroup L of the central of H is invariant under $N(H)$ and under every subgroup of order p^α of G that contains L, and if $N(L)$ has an invariant subgroup of index p, then G has an invariant subgroup of index p. (Received October 26, 1935.)

410. Mr. Alvin Sugar: *A cubic analogue of the Cauchy-Fermat theorem.*

Fermat stated and Cauchy proved the following theorem for polygonal

numbers: every positive integer is a sum of $m+2$ values of $m(x^2-x)/2+x$. Recently, James conjectured the existence of similar theorems for cubic polynomials (American Journal of Mathematics, vol. 56 (1934), p. 305). This paper establishes such a theorem for the cubic polynomials $P(x)=m(x^3-x)/6+x$. The writer evaluated Dickson's constant (Transactions of the American Mathematical Society, vol. 36 (1934), pp. 1–12) and by developing a powerful ascension theorem and ascension methods was able to prove that every positive integer is a sum of $m+3$ values of $P(x)$ for $m \geqq 16$. Since for $m \geqq 3$ there will always exist a positive integer $I(m)$ which is not a sum of fewer than $m+3$ values of $P(x)$, it follows that the above theorems are ideal universal Waring Theorems. (Received October 12, 1935.)

411. Professor H. P. Robertson: *Kinematics and world-structure.*

The idealized cosmological problem, in which nebulae are considered as particles in homogeneous flow, is analyzed from the standpoint of the operational methodology, allowing the fundamental observers the use only of clocks, theodolites, and light-signals. It is found, as an extension of the Helmholtz-Lie solution of the problem of physical space, that such a space-time necessarily admits the introduction of an invariant Riemannian metric of precisely the form and generality of that on which the general relativistic theory of cosmology is based, and in terms of which all given elements can be interpreted in the same way as in the relativistic theory. (Received October 14, 1935.)

412. Professor Morgan Ward: *A calculus of sequences.*

An extensive generalization of portions of the formal parts of algebraic analysis and the calculus of finite differences is obtained by systematically replacing the ordinary binomial coefficient $n(n-1)\cdots(n-r+1)/1\cdot 2\cdots r$ in formulas by the expression $[n, r]=u_n u_{n-1}\cdots u_{n-r+1}/u_1 u_2\cdots u_r$ where $u_0=0, u_1=1, u_2, \cdots, u_n, \cdots$ is a fixed sequence of complex numbers subject to the single restriction $u_n \neq 0$, $n>1$. As an example of the type of results obtained, if the difference $\Delta\phi(x)$ of a polynomial $\phi(x)=\sum_{r=0}^{n} a_{n-r}x^r$ is defined to be $\sum_{r=0}^{n}\sum_{s=1}^{r} a_{n-r}[r, s]x^{r-s}$ and the derivative $D\phi(x)$ is defined to be $\sum_{r=1}^{n} c_{n-r}[r, 1]x^{r-1}$, then symbolically $\Delta=\mathcal{E}(D)-1$ where $\mathcal{E}(t)=1+\sum_1^\infty \alpha t^n/u_1 u_2\cdots u_n$ is the series replacing the exponential $e^t=1+\sum_1^\infty \alpha t^n/1\cdot 2\cdots n$. (Received October 21, 1935.)

413. Professor W. M. Whyburn: *Complexes and manifolds represented by functions of real variables.*

In this paper it is shown that the equation $f(x_1, x_2, \cdots, x_n)=0$, where the real function f is continuous in the real variables $(x_1, \cdots, x_n)$ throughout the interior and boundary of an n-dimensional rectangle H, defines an $(n-1)$-complex within and on the boundary of H provided that at least one point of H satisfies this equation and, furthermore, one of the difference quotients $Q_i=[f(x_1, \cdots, x_{i-1}, x_i', x_{i+1}, \cdots, x_n)-f(x_1, x_2, \cdots, x_n)]/[x_i'-x_i]$ is definite in sign throughout a neighborhood of each point of the locus of this equation

that belongs to H and its boundary. A maximal connected subset of the above described locus which is interior to H is shown to be an $(n-1)$ closed manifold in the analysis situs sense (see Veblen, Colloquium Publications, Volume V, Part II, page 92). The locus defined by k equations of the foregoing type is considered and this locus is shown to be an $(n-k)$ complex when hypotheses of the above type are met by certain kth order difference Jacobians. The paper makes use of a general implicit function theorem that is due to Hedrick and Westfall (Bulletin de la Société Mathématique de France, Vol. 44, pp. 1–14). (Received October 28, 1935.)

414. Professor Clifford Bell: *On a determinant function involving the parameter of a plane curve.*

The function $H_{ij}(t)$ is defined as the determinant $|f_i(t)\ g_j(t)|$ where the subscripts in the determinant indicate the order of the derivatives. The properties of this function are developed, especially those of geometric significance, where $f(t)$, $g(t)$ are taken as the coordinates of any point on an analytic curve. It is shown that $H_{ij}(t)$ may be used in finding inflection points and those multiple points at which two or more tangent lines are coincident. (Received October 29, 1935.)

415. Professor E. T. Bell: *Distributivity of polynomial compositions.*

All polynomial solutions $P(u, v)$, $S(u, v)$ of the functional equations for the associativity and distributivity (right, or left, or both) of P, S are determined, where u, v are elements of a domain of integrity (commutative ring with no nilfactors). This problem arises in the theory of numerical functions. There are precisely twelve solutions. Among the solutions, the fields constitute a three-parameter family. Commutativity of P, S is not assumed. (Received October 31, 1935.)

416. Mr. D. L. Webb: *Definition of Post's generalized negative and maximum in terms of one binary operation.*

In volume 43, pages 163–185 of the American Journal of Mathematics, E. L. Post demonstrates that it is possible to construct a function for any order table in a system of m truth-values by the use of two primitive functions which are generalizations of the negative and maximum in two-valued logic. In this paper Post's generalized negative and maximum are defined in terms of one binary operation. (Received Oct. 31, 1935.)

417. Professor Harry Bateman: *Two systems of polynomials.*

A study is made of the polynomials whose generating function is a power of $1-t^2$ multiplied by a sine or cosine of $2xt(1-t^2)^{-1/2}$. Alternative generating functions are found with the aid of generalized hypergeometric functions and by using the set of generating functions thus extended numerous recurrence relations are obtained. (Received November 1, 1935.)

418. Mr. I. E. Highberg: *A note on abstract polynomials in complex spaces.*

The definition of polynomials in an "espace algébrophile" given by Fréchet is compared with the definition considered by Martin and Michal. It is to be noted that Fréchet's definition is essentially that a function is a polynomial if it is continuous, and if, for some integer n, the $n+1$*th* difference of the function vanishes identically. On the other hand Martin defines $p(x)$ to be a polynomial if it is continuous and if $p(x+\mu y)$ is a polynomial in μ with coefficients depending on x and y. The main result of this note is to show that in complex "espaces algébrophiles" the two definitions are equivalent if we add the requirement of Gateaux differentiability *everywhere* to Fréchet's conditions. (Received November 1, 1935.)

419. Dr. R. D. James: *The number of representations of an integer as a sum of six, ten, and fourteen squares.*

Let $N(2^\alpha m, s)$ denote the number of representations of an integer $2^\alpha m$, $\alpha \geqq 0$, m an odd integer >0, as a sum of s squares. Let $\xi_r(m)$ denote the excess of the sum of the rth powers of the divisors of m which are $\equiv 1 \pmod 4$ over the sum of the rth powers of the divisors of m which are $\equiv 3 \pmod 4$. The following results are proved in this paper: (1) $N(2^\alpha m, 6) = 4\{(-1)^{(m-1)/2}2^{\alpha+2} - 1\}\xi_2(m)$, $m \equiv 1$ or $3 \pmod 4$; (2) $5N(2^\alpha m, 10) = -4(2^{\alpha+4}-1)\xi_4(m)$, $m \equiv 3 \pmod 4$; (3) no relations of the form $N(2^\alpha m, 10) = c(\alpha)\xi_4(m)$, $m \equiv 1 \pmod 4$ or $N(2^\alpha m, 14) = d(\alpha)\xi_6(m)$, $m \equiv 1$ or $3 \pmod 4$ are possible with $c(\alpha)$ and $d(\alpha)$ independent of m. The results for representations by six squares and ten squares have been stated before in different forms (for reference see L. E. Dickson, *History of the Theory of Numbers*, vol. II, chapter IX) but the method of proof here presented is believed to be new. The result for fourteen squares is new. (Received November 1, 1935.)

420. Mr. Wm. A. Mersman: *Abstract integration.*

This is a continuation of a previous paper (see this Bulletin, abstract 40-11-382). The definition of integrability is generalized in such a way that no special treatment is required for unbounded functions or for sets of infinite measure (see also Garrett Birkhoff, Transactions of this Society, vol. 38 (1935), pp. 357–378). It is shown that this integral is included in that of Birkhoff, while, conversely, if his "unconditional convergence" is equivalent to absolute convergence, then his integral is included in this. The present approach to the subject seems to be just as powerful and admits of much simpler proofs. By means of a known method (Bulletin of this Society, vol. 37 (1931), p. 561) the validity of Egoroff's theorem is established for such spaces and functions and is even extended to the case of functions of two continuous variables instead of one continuous and one integral variable. By using this generalization of Egoroff's theorem, the extension of Lebesgue's general convergence theorem is obtained. (Received November 1, 1935.)

421. Professor A. D. Michal and Mr. V. Elconin: *Differential properties of abstract transformation groups with abstract parameters.*

For any x, α in domains D, Δ of two Banach spaces let $f=f(x, \alpha)$ be in D. It is shown in this paper that if the transformations $y=f(x, \alpha)$ form a differentiable group, then for any x, α in D, Δ we have $d_\xi^a f=U(f, V(f, \Omega(\alpha, \xi)))$, where the left member is the Fréchet differential of f with the increment indicated, and the functions $U(x, y)$, $V(x, \alpha)$, $\Omega(\alpha, \beta)$ are linear in their second arguments. It is further shown that under certain hypotheses on U, V, Ω, the solutions $f(x, \alpha)$ of this differential equation which satisfy $f(x, 0)=x$ generate a differentiable transformation group. Hence two fundamental theorems in the classical theory of Lie are generalized to abstract transformation groups with abstract parameters. (Received November 1, 1935.)

422. Professor A. D. Michal and Mr. D. H. Hyers: *The solution of a second order differential system in general analysis as a function of the boundary values.*

This paper is a sequel to a previous one (see abstract 41-9-310) wherein the existence of the solution of $d^2x/dt^2=F(t, x, dx/dt)$ subject to a two-point condition was proved, x being a variable in a Banach space. Here the solution is shown to be continuous and under certain conditions differentiable in the boundary values. As an instance, a simple differential equation in a normed ring is considered. Applications are made to systems of integro-differential equations and to the differential geometry of certain function spaces. (Received November 1, 1935.)

423. Professor A. D. Michal and Mr. E. W. Paxson: *Maps of abstract topological spaces in Banach spaces.*

Homeomorphic maps of topological neighborhoods are made on regions in Banach spaces and in subsets of these spaces. The topological space is metrized through the norm map in the Banach space B. With suitable restrictions placed on the mappings, for example, that the maps in B of the neighborhoods be open sets, bounded or not, and that the maps be of given class, some of the analysis of the topological space may be discussed. (Received Oct. 30, 1935.)

424. Mr. A. E. Taylor: *Analytic functions in general analysis.* Preliminary report.

As has been remarked by Wiener, it is not difficult to show that the ordinary theory of analytic functions of one or more complex variables may be extended to the case that the values of the functions lie in a complete vector space. With this in mind the author has defined analytic functions on one (complete) vector space to another, in such a way that the above theory plays a fundamental role. The resulting theory has been developed from the Cauchy point of view. Results thus far obtained include the following: An analytic function has

Gateaux differentials of all orders, which are analytic, and linear in the increments; all of these functions have an integral representation as in the classical theory. Liouville's theorem is valid, and so also is the Cauchy-Taylor expansion theorem, when suitably expressed in terms of differentials. (Received November 1, 1935.)

425. Mr. Newman A. Hall: *A new class of functions of two variables involving Bessel functions of half an odd integer.*

Functions $C_n(a, b)$ with the generating function $\cos [a(1+x^2)^{1/2}-bx]$ arise in the evaluation of certain integrals connected with the theory of the conduction of heat. These are discussed together with three other sets of functions generated by associated expansions. All these functions satisfy the telegraphic equation in addition to several equations of mixed differences. Finite expressions in terms of Bessel functions of half an odd integer, and several integral representations are given. Certain infinite integrals involving these functions are evaluated. (Received November 2, 1935.)

426. Mr. Alvin Sugar: *On Dickson's method of ascension.*

In a previous paper by the author (to appear in this Bulletin) there is displayed a fundamental set of equations connecting $a=2^8$, $b=3^8, \cdots, e=6^8$. From the existence of this set one is able to conclude that every positive integer is a sum of 566 eighth powers. It is of interest to examine whether there would always exist a finite fundamental set of equations for the case of nth powers and to investigate how such a set could be obtained. Dickson has indicated the solution to these problems in his papers on seventh (American Mathematical Monthly, vol. 41 (1934), pp. 547–555) and ninth powers (Bulletin of this Society, vol. 40 (1934), pp. 487–493). This note offers a detailed and elementary solution to these problems with particulars for eighth powers. (Received October 28, 1935.)

427. Professor Raymond Garver: *Postulates for special types of groups.*

In vol. 40, pp. 698–701, of this Bulletin, I presented a three-postulate definition of group. I now consider simplifications which can be effected in this definition when it is applied to finite or commutative groups, an appropriate postulate being added, of course, in either case. Several results are obtained and and compared with results of other writers. (Received October 30, 1935.)

428. Professor E. T. Bell: *A functional equation in arithmetic.*

It is proved that the only polynomial solutions of the functional equation for associativity are two non-symmetric solutions and a certain symmetric bilinear solution. Hence, the only *polynomial* compositions of numerical functions satisfying the five postulates of D. H. Lehmer (Transactions of this Society, vol. 33, 1931, pp. 945–957) are Cauchy multiplication and Dirichlet multiplication. (Received October 31, 1935.)

429. Professor E. T. Bell: *The reciprocal of a numerical function.*

An immediate proof is given of the author's theorem (Tôhoku Mathematical Journal, vol. 17, 1920, pp. 221–231) that $fg=h$, where f, g, h are numerical functions, and fg is the Dirichlet product of f, g, has a unique solution g when f, h are given and $f(1)\neq 0$. The new proof gives a simpler expression for g as a function of f, h. (Received October 31, 1935.)

430. Professor B. C. Wong: *On certain varieties whose curve sections are hyperelliptic curves.*

This note is concerned with two algebraic varieties whose curve sections are shown to be hyperelliptic curves. One of these varieties is the V_n^{2n+1} in S_{2n+1} which has already been studied by Babbage and by Wong (Proceedings of the Cambridge Philosophical Society, vol. 27 (1931), pp. 399–403, and American Journal of Mathematics, vol. 56 (1934), pp. 219–224, respectively) and the other is the $V_k^{2n-2k+1}$ in S_n which is the residual intersection of $n-k$ cubic hypersurfaces having in common the intersection M_{n-2}^4 of two quadric hypersurfaces of S_n. A few detailed properties of these varieties are described, especially those of their surface sections some of which are already known. For an account of these latter properties the reader is referred to an article by Castelnuovo (see Rendiconti del Circolo Matematico di Palermo, vol. 4 (1890), pp. 73–88). (Received October 31, 1935.)

431. Mr. I. E. Highberg and Mr. A. E. Taylor: *On postulate systems for normed vector spaces.*

In part I of this paper we present a set of eleven independent postulates for real normed vector spaces, and a set of twelve independent postulates for complex normed vector spaces, the latter set being obtained from the former by addition of the postulate $ab(x+y)=a(bx)+(ab)y$. In this part of the paper equality is dealt with as a primitive relation. In part II we show that the equality of two elements x, y is equivalent to $\|x+-1\cdot y\|=0$. We adopt this as a *definition* of equality and by using this concept we give sets of nine and ten independent postulates for real and complex spaces respectively. (Received November 1, 1935.)

432. Mr. I. E. Highberg and Mr. A. E. Taylor: *An independent set of postulates for abstract linear spaces.*

In this paper we complete the work previously reported to the Society in a previous paper on postulate systems for abstract linear spaces (see abstract 41-3-159). A set of twelve independent postulates has been obtained for linear spaces. Equality is treated postulationally. This set differs essentially from other sets in that the two postulates $a(bx)=(ab)x$, $a(x+y)=ax+ay$ are combined into the single postulate $ab(x+y)=a(bx)+a(by)$. (Received November 1, 1935.)

433. Professor A. D. Michal and Mr. V. Elconin: *Hypercommutative rings.*

If xy is the ring multiplication, then $x \odot y \equiv xy - yx = 0$ is the usual commutativity relation (hypercommutativity of order zero). In the first part of this paper we study rings whose elements satisfy the hypercommutativity relation of order one $x \odot y \odot z = 0$ but not $x \odot y = 0$. By exhibiting examples we show that there exist linear algebras and matric algebras that are hypercommutative of order one but not of order zero and are *not* nilpotent. Other triadic relations satisfied by associative or non-associative rings are also studied. In the second part of the paper we consider normed vector rings and study analytic functions defined by differential equations in such abstract rings. Among other theorems we prove: *If $A(x, f)$ is a power series in each variable for each value of the other, then the differential equation $\delta f(x) = A(x, f)\delta x + \delta x A(x, f)$ with Fréchet differentials is completely integrable in a normed vector ring with a hypercommutativity of order one.* (Received November 1, 1935.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

ANDREWS (F. E.). New numbers. How acceptance of a duodecimal base would simplify mathematics. New York, Harcourt, 1935. 168 pp.

BOULIGAND (G.). Géométrie infinitésimale directe et physique mathématique classique. (Mémorial des Sciences Mathématiques dirigé par H. Villat, No. 81.) Paris, Gauthier-Villars, 1935. 60 pp.

BUBB (F. W.). Descriptive geometry. New York, Macmillan, 1935. 15+234 pp.

CANTELLI (F. P.). See L'INSTITUT HENRI POINCARÉ.

CHAMBRAN (P.). Mathématiques. Tome 2: Calcul vectoriel. Paris, Dunod,1935. 226 pp.

——— See DARIÈS (G.).

COPSON (E. T.). An introduction to the theory of functions of a complex variable. Oxford, Clarendon Press, 1935. 448 pp.

DALAKER (H. H.) and HARTIG (H. E.). The calculus. New York and London, McGraw-Hill, 1935. 8+276 pp.

DARIÈS (G.). Mathématiques. 3e édition revisée par P. Chambran. Tome 1: Analyse. Géométrie. Paris, Dunod, 1935. 502 pp.

DICKSON (L. E.). Researches on Waring's problem. (Carnegie Institution of Washington, Publications, No. 464.) Washington, Carnegie Institution, 1935. 5+257 pp.

DÖLP (H.). Grundzüge und Aufgaben der Differential- und Integralrechnung. Neu bearbeitet von E. Netto. 18te Auflage. Berlin, Topelmann, 1935. 214 pp.

GARNIER (R.). Leçons d'algèbre et de géométrie à l'usage des étudiants des Facultés des Sciences. D'après la rédaction de B. Guendjian. Tome 1. Algèbre linéaire. Homographie. Équations tangentielles. Paris, Gauthier-Villars, 1935. 8+234 pp.

GUENDJIAN (B.). See GARNIER (R.).

HARTIG (H. E.). See DALAKER (H. H.).

HAUSDORFF (F.). Mengenlehre. (Göschens Lehrbucherei, I Gruppe, Reine Mathematik, Band 7.) 3te Auflage. Berlin und Leipzig, de Gruyter, 1935. 307 pp.

L'INSTITUT HENRI POINCARÉ, Annales de. Volume 5, fascicule 1: Considérations sur la convergence dans le calcul des probabilités, par F. P. Cantelli; La théorie projective de la relativité, par J. A. Schouten. Paris, Institut Henri Poincaré, Les Presses Universitaires de France, 1935. 88 pp.

KELLS (L. M.). Elementary differential equations. New York and London, McGraw-Hill, 1935. 12+248 pp.

KLUGE (F.). Aloys Müller's Philosophie der Mathematik und Naturwissenschaft. (Studien und Bibliographien zur Gegenwartsphilosophie, Heft 11.) Leipzig, Hirzel, 1935. 106 pp.

MACLEAN (J.). Descriptive mathematics. Bombay, Calcutta, Madras, and London, Macmillan, 1935. 16+143 pp.

MENGER (K.). Herausgegeben von. Ergebnisse eines mathematischen Kolloquiums. Heft 6, 1933–34. Leipzig und Wien, Deuticke, 1935. 47 pp.

MILLER (N.). A first course in differential equations. New York, London, Oxford University Press, 1935. 148 pp.

MONTIGEL (R.). Die natürlichen Werte der goniometrischen Funktionen Sinus und Cosinus von 0° bis 360° alter Teilung in vier Dezimalstellen. Bandoeng-Java, 1935. 43 pp.

NETTO (E.). See DÖLP (H.).

VAN OS (C. H.). Inleiding tot de functietheorie. Groningen-Batavia, Noordhoff, 1935. 253 pp.

PELSENEER (J.). Esquisse du progrès de la pensée mathématique. Des Primitifs au IX• Congrès International des Mathématiciens. (Bibliothèque Scientifique Belge.) Paris, Hermann, 1935. 160 pp.

SCHOUTEN (J. A.). See L'INSTITUT HENRI POINCARÉ.

SLOBIN (H. L.) and SOLT (M. R.). A first course in calculus. New York, Farrar and Rinehart, 1935. 11+426 pp.

SOLT (M. R.). See SLOBIN (H. L.).

STEFFENSEN (J. F.). Forsikringsmatematik. Copenhagen, Gads, 1934. 485 pp.

TRIPIER (H.). Les logarithmes et les puissances en partant de l'hyperbole. Paris, Vuibert, 1934. 8+50 pp.

VILLAT (H.). See BOULIGAND (G.).

WIJDENES (P.). Lagere algebra. Leerboek voor de akte Wiskunde L. O. Deel II. Vergelijkingen, functies, grafieken en reeksen. Derde Druk. Groningen-Batavia, Noordhoff, 1935. 480 pp.

PART II. APPLIED MATHEMATICS

D'ANCONA (U.). See VOLTERRA (V.).

BLACKETT (P. M. S.). Aperçu général. (Actualités Scientifiques et Industrielles, No. 230; La radiation cosmique, No. 1.) Paris, Hermann, 1935. 23 pp.

——— La méthode de la chambre de C. T. R. Wilson. (Actualités Scientifiques et Industrielles, No. 231; La radiation cosmique, No. 2.) Paris, Hermann, 1935. 25 pp.

——— L'action du champ magnétique terrestre. (Actualités Scientifiques et Industrielles, No. 232; La radiation cosmique, No. 3.) Paris, Hermann, 1935. 20 pp.

——— La perte d'énergie par ionisation. (Actualités Scientifiques et Industrielles, No. 233; La radiation cosmique, No. 4.) Paris, Hermann, 1935. 21 pp.

BOGOLIUBOFF (N.). See KRYLOFF (N.).

BORN (M.). Atomic physics. Authorized translation from the German edition by J. Dougall. London, Glasgow, and Bombay, Blackie, 1935. 12+352 pp.

BOULIGAND (G.), GIRAUD (G.), et DELENS (P.). Le problème de la derivée oblique en théorie du potentiel. (Actualités Scientifiques et Industrielles, No. 219; Exposés de Géométrie publié sur la direction de É. Cartan.) Paris, Hermann, 1935. 78 pp.

CARTAN (É.). See BOULIGAND (G.).

COLE (E. B.). The theory of vibrations for engineers: an intermediate course. London, Lockwood, 1935. 10+263 pp.

CONDON (E. U.) and SHORTLEY (G. H.). The theory of atomic spectra. Cambridge, University Press, 1935. 14+442 pp.

DELENS (P.). See BOULIGAND (G.).

DORNER (A.). Mathematik im Dienste der nationalpolitischen Erziehung mit Anwendungsbeispielen aus Volkswissenschaft, Geländekunde und Naturwissenschaft; ein Handbuch für Lehrer. Herausgegeben im Auftrage des "Reichsverbandes Deutscher mathematischer Gesellschaften und Vereine." Frankfurt a. M., Diesterweg, 1935. 4+118 pp.

DOUGALL (J.). See BORN (M.).

FLEURY (P.). Généralités sur les mesures. Préparation, exécution, interprétation, calculs, unités, législation. (Actualités Scientifiques et Industrielles, No. 236.) Paris, Hermann, 1935. 77 pp.

FLÜGGE (W.). Statik und Dynamik der Schalen. Berlin, Springer, 1934. 240 pp.

GALBRUN (H.). Théorie mathématique de l'assurance maladie. Paris, Gauthier-Villars, 1934. 218 pp.

GIRAUD (G.). See BOULIGAND (G.).

GUÉBEN (G.). Structure nucléaire. (Actualités Scientifiques et Industrielles, No. 247; Exposés de physique moléculaire, No. 10.) Paris, Hermann, 1935. 34 pp.

HOARE (C.). The slide rule and how to use it. Containing full, easy and simple instructions to perform all business calculations with unexampled rapidity and accuracy. (Technical Press Manuals.) London, The Technical Press, 1935. 6+104 pp.

INTERNATIONAL TABLES FOR THE DETERMINATION OF CRYSTAL STRUCTURES. Volume 1: Tables on the theory of groups; volume 2: Mathematical and physical tables. (Also in French and German.) Amsterdam, Noordhollandsche Uitgevers Maatschappij; Berlin, Bornträger; London, Bell; New York, Chemical Catalog Company; Paris, Les Presses Universitaires de France, 1935. 20+692 pp.

JAKOBI (S.). See SCHULTZ (E.).

KATZ (J. R.). Die Röntgenspektrographie als Untersuchungsmethode bei hochmolekularen Substanzen, bei Kolloiden und bei tierischen und pflanzlichen Geweben. Berlin und Wien, Urban und Schwarzenberg, 1934, 333 pp.

KEHRMANN (O.). See SCHULTZ (E.).

KNOBLAUCH (O.) und KOCH (W.). Technisch-physikalisches Praktikum. Berlin, Springer, 1934. 167 pp.

KOCH (W.). See KNOBLAUCH (O.).

KRYLOFF (N.) et BOGOLIUBOFF (N.). Méthodes approchées de la mécanique non-linéaire dans leur application à l'étude de la perturbation des mouvements périodiques et de divers phénomènes de résonance s'y rapportant. Kief, Académie des Sciences d'Ukraine, 1935.

LANCHESTER (F. W.). Relativity: an elementary explanation of the space-time relations as established by Minkowski, and a discussion of gravitational theory based thereon. London, Constable, 1935. 14+222 pp.

LANDUCCI (P. C.). Lo spazio e la fisica moderna. Rome, Editrice Studium, 1935. 218 pp.

LILLY (S. B.). See MILLER (J. A.).

LOTKA (A. J.). Théorie analytique des associations biologiques. Publié sur la direction de G. Teissier. Paris, Hermann, 1934. 45 pp.

MAGIE (W. F.). A source book in physics. (Source Books in the History of the Sciences.) New York and London, McGraw-Hill, 1935. 14+620 pp.

MANGOLD (J. F.). Practical mechanics of motion. New York and London, McGraw-Hill, 1935. 263 pp.

MEISSNER (K. W.). Spektroskopie. (Sammlung Göschen, Band 1091.) Berlin und Leipzig, de Gruyter, 1935. 180 pp.

MILLER (J. A.) and LILLY (S. B.). Analytic mechanics. Revised edition. New York and London, Heath, 1935. 16+309 pp.

MINEUR (H.). Dénombrements d'étoiles. Catalogues d'étoiles. Comparaison des séquences photométriques. (Actualités Scientifiques et Industrielles, No. 225.) Paris, Hermann, 1935. 56 pp.

MÜLLER (H.). Führer durch die Technische Mechanik. Berlin, Springer, 1935. 118 pp.

PAULING (L.) and WILSON (E. B.). Introduction to quantum mechanics, with applications to chemistry. London, McGraw-Hill, 1935. 13+468 pp.

ROCARD (Y.). Propagation et absorption du son. (Actualités Scientifiques et Industrielles, No. 222.) Paris, Hermann, 1935. 63 pp.

SCHULTZ (E.). Mathematische und technische Tabellen für Maschinenbau und Elektrotechnik. Neubearbeitet von S. Jakobi und O. Kehrmann. 19te Auflage. Essen, Baedeker Verlag, 1934. 784 pp.

SHORTLEY (G. H.). See CONDON (E. U.).

SITTE (K. F.). Mehrstufige Rahmenformeln: Flotte Verfahren und bequeme Formeln für Zehntausande von praktisch Fällen nebst der allgemeine Grundformeln, eine Anleitung, Zahlenbeispielen und Quellenangaben. Band 1. Brünn, Rohrer Verlag, 1934. 238 pp.

TEICHMANN (H.). Einführung in die Quantenphysik. (Mathematisch-physikalische Bibliothek, Reihe 2, Band 13.) Leipzig und Berlin, Teubner, 1935. 6+93 pp.

TEISSIER (G.). See LOTKA (A. J.).

THOMSON (J.). An introduction to atomic physics. London, Methuen, 1935. 9+228 pp.

THURSTONE (L. L.). The vectors of mind. Multiple-factor analysis for the isolation of primary traits. Chicago, University of Chicago Press, 1935. 15+266 pp.

TOURRIOL (J. B.). Chaleur. (Cours de Physique. Classes de Mathématiques spéciales.) Paris, Gauthier-Villars, 1935. 304 pp.

ULLER (K.). Das Grundgesetz der Wellenfortpflanzung aus bewegter Quelle in bewegtem Mittel. Der Michelson-Versuch und die Raumzeitlehre von Einstein. München und Berlin, Oldenbourg, 1935. 138 pp.

VAN UVEN (M. J.). Mathematical treatment of the results of agricultural and other experiments. Groningen-Batavia, Noordhoff, 1935. 217 pp.

VOLTERRA (V.) et D'ANCONA (U.). Les associations biologiques au point de

vue mathématique. (Actualités Scientifiques et Industrielles, No. 243; Exposés de biométrie et de statistique biologique, No. 5.) Paris, Hermann, 1935. 97 pp.

WECKERING (R.). Stéréophysique: Nouvelles théories sur la constitution de la matière et l'origine des rayonnements; structure dans l'espace des édifices atomiques et moléculaires; processus physiques à l'origine des spectres de raies. Paris, Dunod, 1935. 662 pp.

WILSON (E. B.). See PAULING (L.).

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

LEXINGTON, KENTUCKY, November 29–30, 1935.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 1. All these will appear in the November issue of this BULLETIN. By invitation of the Program Committee, addresses will be delivered by Professor L. L. Dines on *Convex domains and linear inequalities*, and by Professor C. G. Latimer on *The arithmetic of generalized quaternions*.

CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA, November 30, 1935.

Abstracts must be in the hands of Associate Secretary T. M. Putman, University of California, Berkeley, California, not later than November 2. All these will appear in the November issue of this BULLETIN.

ST. LOUIS, MISSOURI, ANNUAL MEETING, December 31, 1935–January 3, 1936.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 26, 1935. Abstracts received by the secretaries by November 2 will appear in the November issue of this BULLETIN. This meeting will be held in conjunction with meetings of the A.A.A.S. and the Mathematical Association of America. By invitation of the Program Committee, addresses will be delivered by Professor Szegö on *Some recent investigations concerning the sections of trigonometric and related series*, and by Professor Synge on *Tensorial methods in dynamics*. On Tuesday afternoon, at a joint session with the Mathematical Association and Section A of the A.A.A.S., Professor Carmichael will deliver his retiring address as Chairman of Section A, on *Linear differential equations of infinite order*. The twelfth Josiah Willard Gibbs Lecture will be delivered by Professor Vannevar Bush on the topic *Mechanical analysis*.

NEW YORK CITY, February 29, 1936.

Abstracts must be in the hands of Associate Secretary J. R. Kline, Low Memorial Library Building, Columbia University, New York City, not later than February 1, 1936.

R. G. D. RICHARDSON, *Secretary of the Society*.

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450–459 Ahnaip St., Menasha, Wis., or Low Memorial Library, Columbia University, New York City.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1934, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, Low Memorial Library, Columbia University, New York City.

WHOLE NUMBER 434

CONTENTS

PAGE

The Summer Meeting at Ann Arbor. By R. G. D. RICHARDSON 753

The Topological Congress in Moscow. By A. W. TUCKER. 764

The Scientific Congress in Mexico. By E. V. HUNTINGTON 765

Baker on Geometry. By T. R. HOLLCROFT 768

Shorter Notices 773

Notes 777

Abstracts of Papers 782

New Publications 804

For official Communications and Notices, see the inside of the back cover.

GEORGE BANTA PUBLISHING COMPANY, MENASHA, WISCONSIN